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A BUCKLING LOAD FORMULA FOR SIMPLY SUPPORTED  
PLATES UNDER COMBINED AXIAL LOAD AND NORMAL PRESSURE

by

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May, 1965

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Submitted to the Department of Naval Architecture and Marine Engineering on May 20, 1965, in partial fulfillment of the requirements for the Master of Science degree in Naval Architecture and Marine Engineering and the Professional degree, Naval Engineer.

ABSTRACT

This is a theoretical study to determine a practical design formula for the buckling load of simply supported rectangular plates under combined axial load and normal pressure.

The theory and method used was based on Samuel Levy's integrated solution of the non-linear differential equations as derived by von Kármán. The length-to-width ratio ( $a/b$ ) of the plates considered was extended up to a value of 4:1. For each length-to-width ratio a set range of normal pressures was used in obtaining the different buckling loads.

The study confirmed the conclusion arrived at by Levy; that is, the normal pressure increases the buckling load. It was further observed and concluded that for low values of  $a/b$  (less than 1.0) the buckling loads obtained in the manner described in Chapter III are impractical to attain. Yield failure will probably occur before the calculated critical load is reached.

In general, however, the authors' recommended buckling load formula for a steel plate is of the form,

$$\sigma_c = \frac{\pi^2 E}{12(1-\mu^2)} \left(\frac{h}{b}\right)^2 K$$

where the value of  $K$  is scaled from Fig. XXXV. The basic limitation on the accuracy of this value of  $K$  is the approximated expression(s) of the deflection equation. The computed value of  $\sigma_c$  must be compared with the yield strength of the material since the validity of the theory only holds true within the elastic region.

Thesis Supervisor: J. Harvey Evans

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SUPPORTED PLATES UNDER COMBINED  
AXIAL LOAD AND NORMAL PRESSURE

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$$P_{cr} = \frac{K \pi^2 E t^3}{12(1-\nu^2) a^3} \left( 1 + \frac{1}{2} \frac{P_{ax}}{P_{cr}} \right)$$

where the value of K is scaled from Fig. XXV. The basic limitation  
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## I. INTRODUCTION

### Background

The discussion of buckling of plates is usually based upon a linear differential equation derived under the assumption that the deflections of the plates are small in comparison with their thickness. In particular, present design practice on hull plating under combined normal pressure and axial load is often based on approximate critical buckling load formulas. The normal load has generally been relegated to minor roles. The reasons purported to support this were very well founded, both from the standpoints of theory and practice. Samuel Levy and others [2], [3] arrived at the conclusion that normal pressure increases the buckling load and thereby indicated that it would be conservative design to neglect the effect of normal pressure. The same conclusion was stated by Bleich [1]. Bleich further made mention of the fact that deflection of ship's hull plating usually does not exceed one-half the plate's thickness and therefore concluded that linearized theory could be applied.

It is worthy to note at this point, however, that deflections exceeding half the plate's thickness are possible, especially in the case of simply supported plates. In those cases linear theory of plates no longer applies. As will be briefly explained in the subsequent paragraph, the problem becomes a non-linear stress problem since deflections of the order of magnitude of the plate thickness must be considered.

Briefly stated, the nonlinearity of the differential equations has its origin in the fact that, in the case of large deflections, there is an interaction between the membrane stresses and the curvature of the plate. This interaction leads to non-linear terms in the equations of



## 1. INTRODUCTION

Abstract

The assumption of small deflections is usually based upon a linear differential equation which is based upon the assumption that the deflection of the plate is small in comparison with its thickness. In particular, present design practice usually assumes that the deflection is small and that the load is often based on approximate critical buckling load formulas. The present study has generally been related to minor cases. The present study is reported to appear to be very well founded, based on the assumptions of theory and practice. General theory and others [1], [2] arrived at the conclusion that normal pressure increases the buckling load and thereby indicates that it would be conservative design to neglect the effect of normal pressure. The same conclusion was stated by Bleich [3]. This further made mention of the fact that deflection of a plate with fixed edges was not fixed one-half the plate thickness and therefore concluded that linearized theory could be applied.

It is to be noted at this point, however, that deflection exceeding half the plate thickness is possible, especially in the case of simply supported plates. In these cases linear theory of plates no longer applies. As will be briefly explained in the subsequent paragraph, the problem becomes a non-linear stress problem since deflections of the order of magnitude of the plate thickness must be considered.

Briefly stated, the nonlinearity of the differential equations has its origin in the fact that, in the case of large deflections, there is an interaction between the moments and stresses and the curvature of the plate. This interaction leads to non-linear terms in the equations of



equilibrium of the plate elements.

The complete differential equations of the problem were formulated by von Karman, who added those non-linear terms pertaining to the flexural rigidity of the plate. Timoshenko [6], Marguerre and Trefftz [8] derived the expressions for the strain energy of plates with large deflections.

An attempt to develop the large deflection theory for rectangular plates under combined bending and longitudinal compression was made by Bengston [7]. His studies included plates with simply supported and clamped edges. However, Bengston introduced, in the course of his analysis, certain arbitrary assumptions which in part contradict each other and it is very doubtful whether the results of his analysis can be considered as entirely correct.

In a series of papers Samuel Levy gave exact theoretical solutions for rectangular plates with large deflections. The theories developed include simply supported and clamped plates.

The problem of rectangular plates carrying longitudinal compression and normal pressure is of prime importance in the design of the hull plating of ships. It is closely related to the question of buckling strength of plates.

#### Statement of the Problem

It is common knowledge that factors of safety are actually factors of ignorance which in some cases have remained unchanged for a period of time for no obvious reason. Factors of safety may be used to account for one or more of the following: (a) material imperfections, (b) faulty construction practices, (c) material degradations due to corrosion, and (d) lack of ability to make rigorous calculations. The conservativeness of design as previously mentioned prompted the authors to ask in what way would conservative design affect the factor of safety; that is, does conservative design actually compound the factor of safety? If, for instance the safety factor is increased by only a few tenths of a percent, then one is justified in using linear theory. However, if the increase becomes considerable, the design may become uneconomical.

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With the salient points stated in the previous paragraph in mind and also with the conclusion of Levy on the effect of normal pressure on the buckling load of plates, the authors came to the conclusion that there is a need for further investigations. First, these investigations afford the possibility of checking the accuracy of simpler approximate methods and formulas. Secondly, and actually the ultimate goal of this thesis, these investigations would hopefully lead to a formulation of a simple design formula for plates subjected to the simultaneous action of normal and edge loads.

A literature survey disclosed there is no simple formula which a designer can use with great facility. Fortunately, general solutions to the differential equations are available in the form of Fourier series. Though the analysis of these solutions is highly involved and the numerical work for obtaining special solutions is very laborious, a method (Chapter III) has been devised which lends itself to computer programming.

This thesis is concerned mainly with plates simply supported on all edges and subjected to combined loadings of uniform normal pressure and axial load in one direction. It is further hoped that it will serve as a forerunner of later investigations based on different boundary conditions.

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A literature survey showed that there are no simple formulas which a designer can use with great facility. Furthermore, several solutions to the differential equations are available in the form of Fourier series. Though the analysis of these solutions is highly involved and the numerical work for obtaining special solutions is very tedious, a method (Chapter III) has been devised which lends itself to computer programming.

This thesis is concerned mainly with plates simply supported on all edges and subjected to constant loading of uniform normal pressure and axial load in one direction. It is further hoped that it will serve as a forerunner of later investigations dealing with different boundary conditions.

## II. THEORY

Nomenclature (See Fig. I):

- $a$  plate length in the x-direction.  
 $b$  plate width in the y-direction.  
 $\alpha = a/b$   
 $h$  plate thickness.  
 $w$  plate deflection.  
 $x, y$  coordinate axes with origin at corner of plate.  
 $E$  Young's modulus.  
 $\mu$  Poisson's ratio. (Note: Tabulated results and figures are for steel with  $\mu = 0.300$ )  
 $p$  uniform normal pressure on plate.  
 $e$  average compressive strain at edges  $y = 0, b$ .  
 $P$  axial load on plate.  
 $F$  stress function.  
 $D = Eh^3/12 (1 - \mu^2)$ , Flexural rigidity of the plate.

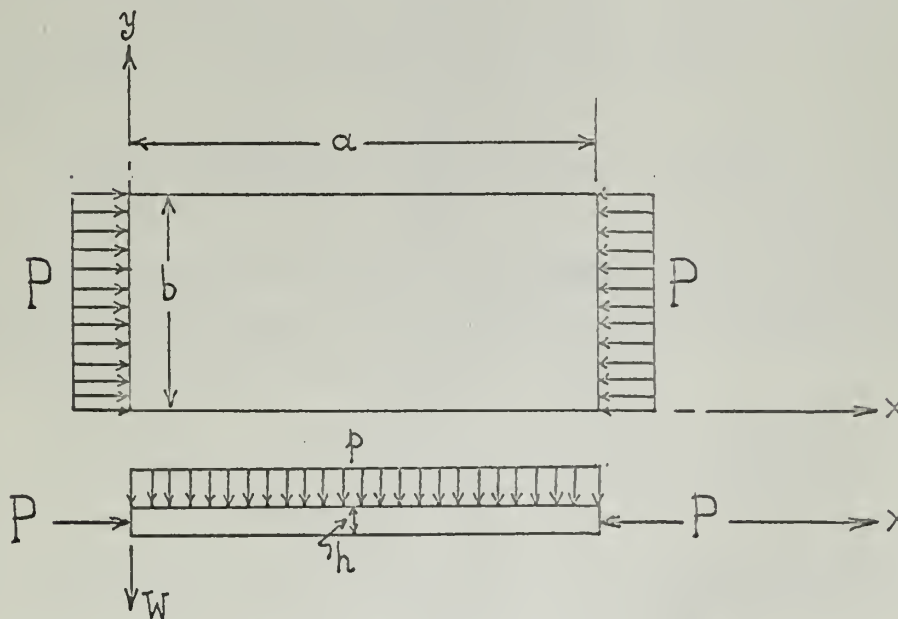


Fig. I Schematic diagram of plate under axial load and normal pressure.





### Fundamental Equations

The fundamental partial differential equations governing the deformations of flat plates have been derived by von Karman. They are given by Samuel Levy [2] and Timoshenko [6]. The exact mathematical analysis to determine the buckling strength of a simply supported flat plate under combined edge compression and normal loading involves the integration of these equations; viz.,

$$\frac{\partial^4 F}{\partial x^4} + 2 \frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = E \left[ \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right] \quad (1)$$

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{D} + \frac{h}{D} \left( \frac{\partial^2 F}{\partial y^2} \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} \right) \quad (2)$$

The boundary conditions for a simply supported rectangular plate to be satisfied by equations (1) and (2) are for deflection,  $w$ , and edge bending moments per unit length to be zero at the edges of the plate; viz.,

$$m_x = -D \left( \frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right) = 0 \quad \text{at } x = 0, x = a \quad (3)$$

$$m_y = -D \left( \frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2} \right) = 0 \quad \text{at } y = 0, y = b \quad (4)$$

The complete solution is more fully explained in reference [2] and, without loss of generality, the deflection equation is here only reproduced in the following Fourier series,

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{m,n} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (5)$$

where the undetermined constants,  $w_{m,n}$ , must satisfy the relation(s) expressed by equation (9) of the same reference.

The average compressive strain,  $e$ , at the edges  $y = 0, b$  is computed from equation (11) of [2] as:

$$e = \frac{P}{Ebh} + \frac{\pi^2}{8\alpha^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m^2 w_{m,n}^2 \quad (6)$$

The fundamental partial differential equations governing the deformation of flat plates have been derived by von Karman. They are given by Samuel Levy (1) and Timoshenko (2). The exact mathematical analysis to determine the bending strength of a simply supported flat plate under combined edge compression and normal loading involves the integration of these equations; viz.,

$$(1) \quad \left[ \frac{\partial^4 F}{\partial x^4} + 2 \frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} \right] = \frac{1}{D} \left[ \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} \right] + \frac{1}{D} \left[ \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} \right]$$

$$(2) \quad \left[ \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right] + \frac{1}{D} \left[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right] = \frac{1}{D} \left[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right] + \frac{1}{D} \left[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right]$$

The boundary conditions for a simply supported rectangular plate to be satisfied by equations (1) and (2) are for deflection,  $w$ , and edge bending moments per unit length to be zero at the edges of the plate; viz.,

$$(3) \quad w = 0 \quad \text{at } x = 0, x = a, y = 0, y = b$$

$$(4) \quad \frac{\partial w}{\partial x} = 0 \quad \text{at } x = 0, x = a, \quad \frac{\partial w}{\partial y} = 0 \quad \text{at } y = 0, y = b$$

The complete solution is more fully explained in reference (2) and, without loss of generality, the deflection equation is here only reproduced in the following Timoshenko notation.

$$(5) \quad w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

where the undetermined constants,  $w_{mn}$ , must satisfy the relation(s) expressed by equation (6) of the same reference.

The average compressive strain,  $\epsilon$ , at the edges  $x = 0, b$  is

computed from equation (1) of (1) as:

$$(6) \quad \epsilon = \frac{1}{Eh} \left[ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{m\pi}{a} w_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \right]_{x=0, b}$$



In this thesis, the deflection equation for large values of  $a/b$  ( $\alpha \gg 1$ ) is approximated by the following expression,

$$w = w_{1,1} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} + w_{3,1} \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b} \\ + w_{5,1} \sin \frac{5\pi x}{a} \sin \frac{\pi y}{b} + w_{2,1} \sin \frac{7\pi x}{a} \sin \frac{\pi y}{b} , \quad (7)$$

and for  $a/b$  - values in the vicinity of 1 and less than 1, the approximated deflection equation is,

$$w = w_{1,1} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} + w_{1,3} \sin \frac{\pi x}{a} \sin \frac{3\pi y}{b} \\ + w_{3,1} \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b} + w_{3,3} \sin \frac{3\pi x}{a} \sin \frac{3\pi y}{b} \\ + w_{1,5} \sin \frac{\pi x}{a} \sin \frac{5\pi y}{b} + w_{5,1} \sin \frac{5\pi x}{a} \sin \frac{\pi y}{b} . \quad (8)$$

There are no special reasons for using two deflection equations for different ranges of  $\alpha$  values, except that equation (7) restricts the deflected shape of the plate to one sine wave across the width and a combination of four sine waves along its length. The errors involved by this approximation are expected to be less than five percent [2], [3]. For this reason, equation (7) could be reduced to a fewer number of terms as  $\alpha$  is reduced. This is evidenced by the deflection equation used for  $\alpha = 3.0$  in reference [2]. More specifically, the contribution of the higher-ordered deflection coefficients in the  $x$ -direction becomes less significant as  $\alpha$  is reduced from an  $\alpha \gg 1$ .

However, as  $\alpha$  is further reduced the assumption of one sine wave across the width of the plate becomes incorrect since the higher-ordered deflection coefficients in the  $y$ -direction become increasingly significant and cannot be neglected unless the desired degree of accuracy is sacrificed. Thus, for such  $\alpha$  values, equation (8) is used. As before, it is believed that the errors incurred by using a finite number of deflection coefficients would give results within the desired accuracy (see Table 13 of [2]).

In some cases, the deflection equation for large values of  $\alpha$  (6) is approximated by the following expression

$$w = w_{1,1} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} + w_{2,1} \sin \frac{2\pi x}{a} \sin \frac{\pi y}{b} + w_{3,1} \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b}$$

$$(7) \quad + w_{2,2} \sin \frac{\pi x}{a} \sin \frac{2\pi y}{b} + w_{3,2} \sin \frac{2\pi x}{a} \sin \frac{2\pi y}{b} + w_{4,2} \sin \frac{4\pi x}{a} \sin \frac{2\pi y}{b}$$

and for  $\alpha$  - values in the vicinity of 1 and less than 1, the approximated deflection equation is

$$w = w_{1,1} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} + w_{2,1} \sin \frac{2\pi x}{a} \sin \frac{\pi y}{b} + w_{3,1} \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b}$$

$$+ w_{2,2} \sin \frac{\pi x}{a} \sin \frac{2\pi y}{b} + w_{3,2} \sin \frac{2\pi x}{a} \sin \frac{2\pi y}{b} + w_{4,2} \sin \frac{4\pi x}{a} \sin \frac{2\pi y}{b}$$

$$(8) \quad + w_{1,3} \sin \frac{\pi x}{a} \sin \frac{3\pi y}{b} + w_{2,3} \sin \frac{2\pi x}{a} \sin \frac{3\pi y}{b} + w_{3,3} \sin \frac{3\pi x}{a} \sin \frac{3\pi y}{b}$$

There are no special reasons for using two deflection equations for different ranges of  $\alpha$  values, except that equation (7) restricts the deflected shape of the plate by one wave across the width and a combination of four sine waves along its length. The errors involved by this approximation are expected to be less than five percent [2]. For this reason, equation (7) could be reduced to a lower number of terms as  $\alpha$  is reduced. This is evidenced by the deflection equation used for  $\alpha = 3.0$  in reference [2]. More specifically, the contribution of the higher-order deflection coefficients in the  $x$ -direction becomes less significant as  $\alpha$  is reduced from  $\alpha = 1$ .

However, as  $\alpha$  is further reduced the assumption of one sine wave across the width of the plate becomes incorrect since the higher-order deflection coefficients in the  $y$ -direction become increasingly significant and cannot be neglected unless the desired degree of accuracy is sacrificed. Thus, for given  $\alpha$  values, equation (8) is used. As before, it is believed that the errors incurred by using a finite number of deflection coefficients would give results within the desired accuracy (see Table 18 of [2]).

The determination of the value of  $\alpha$  applicable to equation (7) or (8) and the more general case of expanding the work of Levy et al on simply supported rectangular plates within the practical values of  $a/b$  are problems examined in this thesis.

The determination of the value of  $\alpha$  is possible to equation (1) or (2) and the more general case of expanding the work of  $\alpha$  on singly supported rectangular plates within the practical values of  $\alpha$  are problems examined in this thesis.



### III. PROCEDURE

With some additional steps and explanatory comments incorporated, the procedure followed in solving for the deflection equations was essentially the same as that outlined in reference [3]. The steps were as follows:

1. The family of four or six simultaneous cubic equations corresponding to the same number of unknown deflection coefficients was first obtained. This was done by solving for the coefficients  $b_{p,q}$  defined by equation (8) of reference [2], and substituting them into equation (9) of the same reference. The results are shown in Appendix A.

2. Each of the resulting equations was divided by  $h^3$ . This step nondimensionalized the family of equations.

3. Values of  $w_{1,1}/h$ ,  $w_{1,3}/h$ ,  $w_{3,1}/h$ , etc. were estimated corresponding to chosen values of  $Pb/Eh^3$  and  $pb^4/Eh^4$  -- the non-dimensional axial load and normal pressure, respectively. This was the most delicate step since at higher values of  $Pb/Eh^3$  (normal pressure held constant) more than one solution was found possible (see [3] also). In other words, the estimated  $w$ 's, when not properly chosen, could lead to solutions other than those corresponding to a continuous change in buckled form from zero axial load to the buckling load.

It was therefore decided that a step-by-step method of estimating the  $w$ 's be employed. In conjunction with the subsequent steps 4, 5, and 6, this method proceeds as follows:

- a). Corresponding to three low values of  $Pb/Eh^3$ , including  $Pb/Eh^3 = 0$ , three sets of solutions were obtained using a reasonable set of  $w$ -estimates. These values of  $Pb/Eh^3$  were hoped to lead to unique solutions. For these estimated  $w$ 's, reference [2] or [3] was used as a guide.

### III. PROCEDURE

With some additional steps and temporary comments incorporated, the procedure followed in solving for the deflection equations was essentially the same as that outlined in reference [1]. The steps were as follows:

1. The family of four or six simultaneous cubic equations corresponding to the same number of unknown deflection coefficients was first obtained. This was done by solving for the coefficients  $w_1, w_2, w_3, w_4$  defined by equation (8) of reference [1], and substituting them into equation (9) of the same reference. The results are shown in Appendix A.
2. Each of the resulting equations was divided by  $h^3$ . This step nondimensionalized the family of equations.

3. Values of  $w_1, w_2, w_3, w_4$ , etc. were estimated corresponding to chosen values of  $P/Eh^3$  and  $P/Eh^4$  -- the non-dimensional axial load and normal pressure, respectively. This was the most delicate step since at higher values of  $P/Eh^3$  (normal pressure held constant) more than one solution was found possible (see [1] also). In other words, the estimated  $w$ 's, when not properly chosen, could lead to solutions other than those corresponding to a continuous change in buckled form from zero axial load to the buckling load.

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b). From (a) above, succeeding estimates of  $w$ 's as axial load is increased, were based on the continued trend of each  $w$ -vs- $Pb/Eh^3$  curve; viz., the slope and the rate of change of slope computed from the last three correct points were employed in an abridged Taylor series of the form,

$$w(x_0 + h) = w(x_0) + w'(x_0) \frac{h}{1!} + w''(x_0) \frac{h^2}{2!},$$

where,  $w(x_0 + h)$  = next estimate of the particular  $w$ .

$w(x_0)$  = last computed value of  $w$ .

$w'(x_0)$  = slope obtained from the last two computed points.

$w''(x_0)$  = rate of change of slope based on the last three computed values of  $w$ .

4. The resulting cubic equations shown in Appendix A were linearized by expanding the right-hand side of each equation in a Taylor series in the neighborhood of the estimated values of  $w_{1,1}/h$ ,  $w_{1,3}/h$ ,  $w_{3,1}/h$ , etc., omitting higher ordered terms.

5. Crout's method [ 5 ] was used in solving for the difference between the estimated  $w$ 's and their improved values.

6. Step 5 was repeated until such time that the calculated error sum was less than a test constant. The error sum was defined by the authors as the measure of closeness of successive approximations. In equation form, it is as follows:

$$\text{Error} = \sum_{i=1}^N x_i$$

where,  $x_i$  represents the absolute value of the difference between an estimate  $w$  and its improved value.

The test constant used for equation (7) was 0.0001 and for equation (8), 0.000001. Why two different values of the test constant

4). From (2) above, successive estimates of  $w$  as  $w_0, w_1, w_2, \dots$  were based on the continuous trend of  $w$  as  $w_0, w_1, w_2, \dots$  and the rate of change of slope computed from the last three correct points were employed in an extended Taylor series of the form,

$$w(x) = w_0 + w'_0(x - x_0) + \frac{w''_0}{2!}(x - x_0)^2 + \frac{w'''_0}{3!}(x - x_0)^3 + \dots$$

where,  $w_0 = w(x_0)$  = new estimate of the parameter  $w$ .

$w'_0 = w'(x_0)$  = last computed value of  $w$ .

$w''_0 = w''(x_0)$  = slope obtained from the last two computed points.

$w'''_0 = w'''(x_0)$  = rate of change of slope based on the last three computed values of  $w$ .

5). The resulting cubic equation above in Appendix A were linearized by expanding the right-hand side of each equation in a Taylor series in the neighborhood of the estimated values of  $w_0, w'_0, w''_0, w'''_0$ , etc., resulting higher ordered terms.

6). Gauss's method [1] was used in solving for the difference between the estimated  $w$ 's and their improved values.

7). Step 6 was repeated until such time that the calculated error sum was less than a test constant. The error sum was defined by the authors as the measure of closeness of successive calculations.

In equation form, it is as follows:

$$Error = \sum_{i=1}^N \frac{1}{N} \left| \frac{w_i - w_{i-1}}{w_i} \right|$$

where,  $x_i$  represents the absolute value of the difference between an estimate  $w$  and its improved value.

The test constant used for equation (7) was 0.0001 and for equation (5), 0.00001. With two different values of the test constant



were used would become evident from a study of the results in Chapter IV.

7. The average strain corresponding to each axial load was computed from equation (6).

8. In the vicinity of the buckling load where the strain started to change more rapidly than the axial load, the  $w_{1,1}/h$  was made the independent variable in place of  $Pb/Eh^3$ . The other deflection coefficients were computed in a similar fashion as before.

Values of the buckling load were obtained for values of  $pb^4/Eh^4$  equal to 2.50, 7.50, 12.50, 18.00, 24.50, and 30.00, and at values of  $a/b$  up to 4.00. These were plotted versus  $a/b$  and analyzed for a possible comparison with Bryan's classical solution.

The whole procedure(s) had been programmed with the use of the IBM-7094. Details not otherwise covered here may be found in the programs shown in Appendix B.

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#### Chapter IV.

7. The two are strain corresponding to each axial load was computed from equation (3).

8. In the vicinity of the buckling load where the strain started to change more rapidly than the axial load, the  $w_{1,1}$  was made the independent variable in place of  $P/P_{cr}$ . The other deflection coefficients were computed in a similar fashion as before.

Values of the buckling load were obtained for values of  $P/P_{cr}$  equal to 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0, and 11.0. These were plotted versus  $\lambda$  and analyzed for a possible comparison with Bryan's classical solution.

The whole process (a) had been programmed with the use of the IBM-7094. Details not otherwise covered here may be found in the program shown in Appendix B.

#### IV. RESULTS

Tables I to XII are sample results of the computer programs. Tables I to VI were obtained using equation (8) for an  $a/b = 1.00$  and, the rest of the tables were for an  $a/b = 4.00$  using equation (7).

Figures II to XXXIII are the graphical results of the same programs. The curves drawn are the stress-strain curve analogy for plates with the non-dimensional axial load,  $Pb/Eh^3$ , as ordinate and the non-dimensional average strain,  $eb^2/h^2$ , as abscissa. (Note: Not all results for the chosen  $a/b$ -values used were plotted.)

Tabulated values of the critical loads, rounded to the nearest hundredth, are shown in Tables XII and XIV.

Figure XXXIV is a plot of the critical  $Pb/Eh^3$  versus  $a/b$ . The curves between zero  $a/b$  and the smallest value of  $a/b$  used have been extrapolated. The curves have been faired at points where the results of both equations (7) and (8) either become tangent or intersect. Refinements of the curves in the vicinity of the cusps were made by obtaining more results at the  $a/b$ -values concerned. These results are shown in Tables XIII and XIV.



#### IV. RESULTS

Tables I to XII are sample results of the computer programs. Tables I to VI were obtained using equation (6) for  $\alpha/\beta = 1.00$  and the rest of the tables were for  $\alpha/\beta = 0.00$  using equation (7).

Figures II to X, XII are the graphical results of the same programs. The curves shown are the stress-strain curve analysis for plates with the non-dimensional axial load,  $\sigma/\sigma_0$ , as ordinate and the non-dimensional average strain,  $\epsilon/\epsilon_0$ , as abscissa. (Note: Not all results for the chosen  $\alpha/\beta$ -values were plotted.)

Tabulated values of the critical loads, rounded to the nearest hundredths, are shown in Tables XII and XIV.

Figure XIII is a plot of the critical  $P_{cr}/P_0$  versus  $\alpha/\beta$ . The curves between zero  $\alpha/\beta$  and the smallest value of  $\alpha/\beta$  used have been extrapolated. The curves have been failed at points where the results of both equations (7) and (6) either became tangent or intersect. Segments of the curves in the vicinity of the cusps were made by obtaining more results at the  $\alpha/\beta$ -values concerned. These results are shown in Tables XIII and XIV.

Table I - Values of deflection coefficients for various values of axial compressive load in the x-direction,  $P$ , for simply supported rectangular plate,  $a/b = 1.00$ ,  $\mu = 0.300$ .  
Normal pressure,  $p = 2.50 E h^4 / b^4$ .

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{1,3}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{3,3}}{h}$	$\frac{w_{1,5}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{eb^2}{h^2}$
0.	0.113109	0.001522	0.001522	0.000156	0.000134	0.000134	0.015
0.500	0.130955	0.001535	0.001606	0.000159	0.000135	0.000137	0.521
1.000	0.155300	0.001552	0.001704	0.000162	0.000135	0.000140	1.029
1.500	0.190218	0.001580	0.001826	0.000166	0.000135	0.000143	1.544
2.000	0.243439	0.001636	0.001995	0.000170	0.000135	0.000146	2.073
2.500	0.329375	0.001781	0.002298	0.000177	0.000135	0.000150	2.633
3.000	0.466786	0.002213	0.003028	0.000195	0.000136	0.000154	3.268
3.500	0.651813	0.003327	0.004820	0.000246	0.000137	0.000161	4.024
4.000	0.848735	0.005355	0.008196	0.000384	0.000143	0.000176	4.889
4.500	1.034310	0.008183	0.013194	0.000667	0.000154	0.000205	5.821
5.000	1.204318	0.011610	0.019678	0.001151	0.000172	0.000252	6.793
5.500	1.360460	0.015481	0.027524	0.001886	0.000196	0.000320	7.792
6.000	1.505171	0.019690	0.036648	0.002916	0.000227	0.000412	8.810
6.500	1.640526	0.024166	0.046998	0.004283	0.000264	0.000530	9.845
7.000	1.768140	0.028864	0.058539	0.006030	0.000308	0.000674	10.896
7.500	1.889255	0.033752	0.071252	0.008198	0.000359	0.000846	11.961
8.000	2.004827	0.038810	0.085125	0.010828	0.000420	0.001046	13.042
8.500	2.115604	0.044023	0.100152	0.013961	0.000492	0.001273	14.137
9.000	2.222177	0.049379	0.116321	0.017636	0.000580	0.001528	15.248
9.530	2.325014	0.054867	0.133658	0.021890	0.000688	0.001807	16.376
10.000	2.424491	0.060477	0.152133	0.026757	0.000821	0.002110	17.521
10.500	2.520910	0.066194	0.171741	0.032266	0.000986	0.002434	18.684
11.000	2.614510	0.071999	0.192479	0.038442	0.001190	0.002775	19.867
11.500	2.705480	0.077869	0.214327	0.045301	0.001442	0.003128	21.070
12.000	2.793967	0.083776	0.237259	0.052850	0.001751	0.003489	22.295
12.500	2.880085	0.089685	0.261241	0.061089	0.002128	0.003851	23.543
13.000	2.963914	0.095555	0.286231	0.070006	0.002583	0.004206	24.813
13.500	3.045514	0.101342	0.312181	0.079581	0.003127	0.004546	26.108
14.000	3.124924	0.106995	0.339036	0.089786	0.003771	0.004861	27.427
14.500	3.202167	0.112462	0.366738	0.100586	0.004525	0.005139	28.772
15.000	3.277257	0.117688	0.395229	0.111941	0.005399	0.005370	30.142
15.500	3.350196	0.122617	0.424455	0.123810	0.006404	0.005540	31.537
16.000	3.420982	0.127192	0.454368	0.136152	0.007549	0.005637	32.957



16.000	3.450085	0.151165	0.454398	0.136195	0.001246	0.002964	25.021
12.200	3.320106	0.138611	0.757452	0.153810	0.009404	0.002246	31.031
12.000	3.311571	0.111398	0.362556	0.111941	0.006366	0.001230	30.145
14.200	3.205161	0.115495	0.366136	0.103999	0.004272	0.002134	33.115
14.000	3.154934	0.109002	0.329026	0.086136	0.002911	0.000601	31.451
13.500	3.042274	0.101344	0.317121	0.082281	0.003151	0.001546	30.109
13.000	2.964214	0.092422	0.387931	0.090909	0.003223	0.004506	34.813
13.200	2.993082	0.089652	0.521241	0.061966	0.007158	0.003221	35.243
15.000	2.462344	0.083119	0.531494	0.023834	0.001121	0.003479	35.502
14.200	2.462469	0.051820	0.514351	0.042201	0.001445	0.003166	31.010
14.000	2.411240	0.051028	0.135114	0.038443	0.001100	0.003117	16.991
16.900	2.230910	0.006134	0.111441	0.032366	0.001082	0.003434	16.024
16.000	2.434491	0.029011	0.125130	0.036101	0.000731	0.002110	11.131
9.500	2.312014	0.024821	0.133428	0.071600	0.000698	0.001401	19.310
9.000	2.136111	0.049310	0.116321	0.011082	0.000980	0.001452	19.548
8.200	2.112804	0.044013	0.100125	0.013121	0.000485	0.001013	14.121
8.000	2.044621	0.038810	0.082132	0.010276	0.000720	0.001046	13.045
1.200	1.380722	0.033128	0.051122	0.008166	0.002326	0.000416	11.021
1.000	1.368441	0.038224	0.026223	0.002036	0.000302	0.000117	10.806
6.200	1.341732	0.035172	0.042608	0.004882	0.000364	0.000230	8.845
6.000	1.261111	0.012860	0.032247	0.003012	0.000321	0.000412	8.012
2.200	1.290460	0.012491	0.051224	0.001592	0.000106	0.000220	1.152
2.000	1.101218	0.011910	0.017611	0.001121	0.000115	0.000525	2.103
4.200	1.031810	0.048133	0.012121	0.000921	0.000124	0.000769	4.951
4.000	0.818122	0.002224	0.002166	0.000921	0.000143	0.000119	4.866
3.200	0.621813	0.003251	0.004837	0.000018	0.000131	0.000191	4.034
3.000	0.460482	0.005513	0.003053	0.000122	0.000139	0.000124	3.803
2.800	0.252212	0.061121	0.001838	0.000111	0.000122	0.000129	3.012
2.000	0.243120	0.001429	0.001321	0.000110	0.000132	0.000113	1.244
1.200	0.190813	0.001120	0.001104	0.000127	0.000132	0.000110	1.057
1.000	0.172300	0.001221	0.001202	0.000126	0.000122	0.000131	0.921
0.200	0.112122	0.001215	0.001221	0.000122	0.000124	0.000124	0.912

Table 1  
 Values of  $b$ ,  $b' = 5.50 \times 10^{-10}$  m  
 x-direction,  $b'$  for primary absorption coefficient  $\mu_p$ ,  $b' = 1.00 \times 10^{-10}$  m = 0.300  
 Values of reflection coefficients for various angles of axis,  $\theta$  and  $\phi$  in the



Table I (cont.) - Values of deflection coefficients for various values of axial compressive load in the x-direction,  $P$ , for simply supported rectangular plate,  $a/b = 1.00$ ,  $\mu = 0.300$ . Normal pressure,  $p = 2.50 E h^4/b^4$ .

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{1,3}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{3,3}}{h}$	$\frac{w_{1,5}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{eb^2}{h^2}$
16.500	3.489605	0.131359	0.484926	0.148926	0.008843	0.005646	34.402
17.000	3.556047	0.135060	0.516103	0.162099	0.010296	0.005552	35.873
17.500	3.620286	0.138240	0.547881	0.175642	0.011916	0.005341	37.369
18.000	3.682288	0.140841	0.580262	0.189534	0.013714	0.004996	38.891
18.500	3.742007	0.142805	0.613260	0.203761	0.015701	0.004502	40.438
19.000	3.799381	0.144069	0.646908	0.218318	0.017886	0.003841	42.011
19.500	3.854328	0.144565	0.681258	0.233211	0.020285	0.002996	43.611
20.000	3.906737	0.144218	0.716381	0.248451	0.022910	0.001948	45.239
20.500	3.956468	0.142943	0.752370	0.264062	0.025780	0.000677	46.897
21.000	4.003334	0.140647	0.789343	0.280075	0.028913	-0.000837	48.586
21.500	4.047101	0.137221	0.827445	0.296530	0.032333	-0.002613	50.310
22.000	4.087464	0.132543	0.866846	0.313473	0.036063	-0.004672	52.070
22.500	4.124046	0.126482	0.907749	0.330955	0.040133	-0.007032	53.871
23.000	4.156369	0.118897	0.950386	0.349025	0.044572	-0.009707	55.717
23.500	4.183853	0.109654	0.995010	0.367720	0.049409	-0.012703	57.612
24.000	4.205803	0.098647	1.041883	0.387049	0.054667	-0.016014	59.562
24.500	4.221416	0.085840	1.091255	0.406970	0.060358	-0.019621	61.571
25.000	4.229790	0.071321	1.143337	0.427361	0.066472	-0.023485	63.643
25.500	4.229896	0.055357	1.198303	0.447995	0.072967	-0.027555	65.779
26.015	4.219896	0.037904	1.258260	0.469164	0.079986	-0.031938	68.049
26.282	4.209896	0.028733	1.290778	0.479911	0.083709	-0.034291	69.250
26.479	4.199896	0.021994	1.315644	0.487706	0.085491	-0.036091	70.151
26.638	4.189895	0.016648	1.336290	0.493858	0.088743	-0.037593	70.885
26.771	4.179895	0.012261	1.354111	0.498900	0.090636	-0.038904	71.506
26.885	4.169895	0.008598	1.369846	0.503121	0.092259	-0.040080	72.043
26.984	4.159895	0.005512	1.383944	0.506695	0.093668	-0.041154	72.513
27.070	4.149895	0.002903	1.396704	0.509740	0.094900	-0.042148	72.927
27.145	4.139895	0.000699	1.408341	0.512342	0.095983	-0.043078	73.295
27.211	4.129895	-0.001158	1.419012	0.514563	0.096937	-0.043956	73.622
27.270	4.119895	-0.002711	1.428842	0.516454	0.097777	-0.044790	73.914
27.320	4.109895	-0.003996	1.437929	0.518054	0.098517	-0.045587	74.173
27.365	4.099895	-0.005043	1.446354	0.519395	0.099167	-0.046352	74.404
27.403	4.089895	-0.005877	1.454183	0.520505	0.099737	-0.047089	74.609

34.703	4.068909	-0.008841	1.494193	0.230502	0.068191	-0.044092	14.809
31.929	4.068902	-0.009043	1.494324	0.218392	0.068191	-0.040925	14.704
29.159	4.068902	-0.008279	1.434953	0.218054	0.067911	-0.042231	14.113
26.389	4.068962	-0.003511	1.438873	0.219124	0.068111	-0.041100	13.814
23.619	4.068902	-0.004004	1.440013	0.218392	0.068191	-0.043928	13.423
20.849	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	13.127
18.079	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	12.831
15.309	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	12.535
12.539	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	12.239
9.769	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	11.943
6.999	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	11.647
4.229	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	11.351
1.459	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	11.055
-1.311	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	10.759
-4.081	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	10.463
-6.851	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	10.167
-9.621	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	9.871
-12.391	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	9.575
-15.161	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	9.279
-17.931	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	8.983
-20.701	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	8.687
-23.471	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	8.391
-26.241	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	8.095
-29.011	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	7.799
-31.781	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	7.503
-34.551	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	7.207
-37.321	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	6.911
-40.091	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	6.615
-42.861	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	6.319
-45.631	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	6.023
-48.401	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	5.727
-51.171	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	5.431
-53.941	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	5.135
-56.711	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	4.839
-59.481	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	4.543
-62.251	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	4.247
-65.021	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	3.951
-67.791	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	3.655
-70.561	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	3.359
-73.331	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	3.063
-76.101	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	2.767
-78.871	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	2.471
-81.641	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	2.175
-84.411	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	1.879
-87.181	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	1.583
-89.951	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	1.287
-92.721	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	0.991
-95.491	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	0.695
-98.261	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	0.399
-101.031	4.068902	0.003792	1.492941	0.230502	0.068191	-0.044092	0.103

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for  $x = 0$  and  $x = 1$

Table 1 (cont.)



Table I (cont.) - Values of deflection coefficients for various values of axial compressive load in the x-direction, P, for simply supported rectangular plate,  $a/b = 1.00$ ,  $\mu = 0.300$ .  
Normal pressure,  $p = 2.50 Eh^4/b^4$ .

$\frac{Pb^3}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{1,3}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{3,3}}{h}$	$\frac{w_{1,5}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{eb^2}{h^2}$
27.437	4.079895	-0.006518	1.461472	0.521407	0.100233	-0.047802	74.790
27.465	4.069895	-0.006986	1.468271	0.522120	0.100663	-0.048493	74.949
27.489	4.059895	-0.007297	1.474621	0.522662	0.101031	-0.049164	75.088
27.509	4.049895	-0.007465	1.480560	0.523047	0.101344	-0.049818	75.209
27.524	4.039895	-0.007503	1.486119	0.523290	0.101605	-0.050457	75.313
27.537	4.029895	-0.007422	1.491328	0.523401	0.101819	-0.051080	75.402
27.546	4.019895	-0.007231	1.496211	0.523390	0.101989	-0.051691	75.475
27.551	4.009895	-0.006940	1.500794	0.523268	0.102118	-0.052288	75.535
27.554	3.999895	-0.006557	1.505095	0.523043	0.102210	-0.052874	75.582
27.555	3.989895	-0.006090	1.509135	0.522723	0.102266	-0.053449	75.617
27.552	3.979894	-0.005545	1.512931	0.522324	0.102291	-0.054014	75.641





Table II - Values of deflection coefficients for various values of axial compressive load in the x-direction, P, for simply supported rectangular plate,  $a/b = 1.00$ ,  $\mu = 0.300$ .  
Normal pressure,  $p = 7.50 \text{ Eh}^4/b^4$ .

$\frac{Pb}{\text{Eh}^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{1,3}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{3,3}}{h}$	$\frac{w_{1,5}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{eb^2}{h^2}$
0.	0.329069	0.004721	0.004721	0.000485	0.000403	0.000403	0.133
0.500	0.375274	0.004842	0.005064	0.000499	0.000404	0.000412	0.674
1.000	0.433827	0.005031	0.005516	0.000518	0.000404	0.000421	1.232
1.500	0.508237	0.005343	0.006154	0.000545	0.000405	0.000430	1.819
2.000	0.601481	0.005870	0.007116	0.000587	0.000406	0.000441	2.446
2.500	0.714013	0.006741	0.008610	0.000658	0.000408	0.000454	3.129
3.000	0.842155	0.008089	0.010891	0.000781	0.000411	0.000472	3.876
3.500	0.979252	0.009994	0.014177	0.000988	0.000418	0.000496	4.685
4.000	1.118822	0.012457	0.018599	0.001321	0.000429	0.000531	5.548
4.500	1.256523	0.015424	0.024205	0.001821	0.000446	0.000580	6.454
5.000	1.390149	0.018820	0.030995	0.002531	0.000467	0.000646	7.395
5.500	1.518869	0.022574	0.038954	0.003492	0.000494	0.000731	8.363
6.000	1.642574	0.026629	0.048062	0.004746	0.000527	0.000839	9.355
6.500	1.761487	0.030937	0.058305	0.006331	0.000566	0.000970	10.367
7.000	1.875953	0.035466	0.069672	0.008287	0.000612	0.001126	11.397
7.500	1.986346	0.040189	0.082137	0.010655	0.000667	0.001308	12.445
8.000	2.093025	0.045088	0.095759	0.013473	0.000732	0.001515	13.510
8.500	2.196316	0.050148	0.110476	0.016780	0.000809	0.001749	14.593
9.000	2.296504	0.055356	0.126310	0.020613	0.000904	0.002007	15.692
9.500	2.393836	0.060701	0.143262	0.025007	0.001019	0.002290	16.809
10.000	2.488521	0.066170	0.161328	0.029994	0.001160	0.002594	17.944
10.500	2.580733	0.071745	0.180504	0.035602	0.001335	0.002919	19.099
11.000	2.670615	0.077408	0.200779	0.041851	0.001550	0.003260	20.273
11.500	2.758281	0.083135	0.222136	0.048757	0.001813	0.003613	21.469
12.000	2.843819	0.088897	0.244550	0.056327	0.002133	0.003973	22.686
12.500	2.927293	0.094660	0.267990	0.064559	0.002529	0.004334	23.927
13.000	3.008751	0.100383	0.292416	0.073442	0.002985	0.004688	25.190
13.500	3.088220	0.106024	0.317781	0.082957	0.003536	0.005028	26.478
14.000	3.165716	0.111533	0.344034	0.093078	0.004186	0.005344	27.790
14.500	3.241245	0.116862	0.371120	0.103773	0.004942	0.005625	29.127
15.000	3.314803	0.121956	0.398934	0.115003	0.005816	0.005860	30.489
15.500	3.386382	0.126763	0.427576	0.126731	0.006817	0.006037	31.876
16.000	3.455968	0.131229	0.456848	0.138918	0.007954	0.006143	33.289



10.000	3.72200E	0.131553	0.426046	0.182016	0.001024	0.001142	33.580
12.500	3.289385	0.152103	0.451253	0.158131	0.002914	0.002021	31.842
15.000	2.914903	0.171698	0.268094	0.112003	0.002912	0.002880	30.480
17.500	3.541542	0.116905	0.341150	0.103113	0.004045	0.002872	30.171
20.000	3.166419	0.111933	0.344034	0.062019	0.004189	0.002044	31.200
22.500	3.000350	0.106054	0.311481	0.065323	0.002236	0.002059	30.440
25.000	3.066124	0.100583	0.285416	0.043643	0.002062	0.001567	32.150
27.500	2.453143	0.064666	0.264803	0.064229	0.002700	0.002324	31.881
30.000	3.213016	0.062934	0.244226	0.044221	0.001883	0.002022	33.520
32.500	3.123241	0.063122	0.233139	0.044111	0.001013	0.002212	31.400
35.000	2.410519	0.041400	0.240116	0.01821	0.001269	0.002390	30.343
37.500	2.420122	0.041542	0.190204	0.032403	0.001362	0.002010	32.008
40.000	3.407251	0.040410	0.101822	0.056034	0.001100	0.002094	31.541
42.500	3.203530	0.007322	0.110216	0.057001	0.002010	0.002050	30.909
45.000	2.188312	0.020170	0.110441	0.012180	0.002094	0.002001	32.403
47.500	3.032032	0.042988	0.042510	0.012413	0.001683	0.001132	32.710
50.000	1.585446	0.040189	0.041111	0.010022	0.002501	0.001200	32.142
52.500	1.846723	0.034400	0.000512	0.000781	0.000017	0.001797	31.210
55.000	1.101121	0.031021	0.004206	0.004211	0.002093	0.001700	30.721
57.500	1.063544	0.030220	0.000624	0.001007	0.000017	0.000023	32.320
60.000	1.810805	0.004414	0.000624	0.001007	0.000017	0.000023	32.320
62.500	1.101121	0.031021	0.004206	0.004211	0.002093	0.001700	30.721
65.000	1.063544	0.030220	0.000624	0.001007	0.000017	0.000023	32.320
67.500	1.810805	0.004414	0.000624	0.001007	0.000017	0.000023	32.320
70.000	1.101121	0.031021	0.004206	0.004211	0.002093	0.001700	30.721
72.500	1.063544	0.030220	0.000624	0.001007	0.000017	0.000023	32.320
75.000	1.810805	0.004414	0.000624	0.001007	0.000017	0.000023	32.320
77.500	1.101121	0.031021	0.004206	0.004211	0.002093	0.001700	30.721
80.000	1.063544	0.030220	0.000624	0.001007	0.000017	0.000023	32.320
82.500	1.810805	0.004414	0.000624	0.001007	0.000017	0.000023	32.320
85.000	1.101121	0.031021	0.004206	0.004211	0.002093	0.001700	30.721
87.500	1.063544	0.030220	0.000624	0.001007	0.000017	0.000023	32.320
90.000	1.810805	0.004414	0.000624	0.001007	0.000017	0.000023	32.320
92.500	1.101121	0.031021	0.004206	0.004211	0.002093	0.001700	30.721
95.000	1.063544	0.030220	0.000624	0.001007	0.000017	0.000023	32.320
97.500	1.810805	0.004414	0.000624	0.001007	0.000017	0.000023	32.320
100.000	1.101121	0.031021	0.004206	0.004211	0.002093	0.001700	30.721

$\frac{E^2}{J}$        $\frac{E^2}{J^2}$        $\frac{H}{m^2 J}$        $\frac{E^2}{m^2 J}$        $\frac{H}{m^2 J}$        $\frac{H}{m^2 J}$        $\frac{H}{m^2 J}$

Table II - Values of reflection coefficients for various values of cell permeability  $\mu$  and  $\epsilon$ .



Table II (cont.) - Values of deflection coefficients for various values of axial compressive load in the x-direction,  $P$ , for simply supported rectangular plate,  $a/b = 1.00$ ,  $\mu = 0.300$ .  
Normal pressure,  $p = 7.50 E h^4 / b^4$ .

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{1,3}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{3,3}}{h}$	$\frac{w_{1,5}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{e_b^2}{h^2}$
16.500	3.523541	0.135299	0.486762	0.151527	0.009236	0.006165	34.726
17.000	3.589080	0.138921	0.517291	0.164526	0.010672	0.006088	36.188
17.500	3.652556	0.142039	0.548417	0.177889	0.012271	0.005897	37.676
18.000	3.713931	0.144599	0.580139	0.191594	0.014042	0.005579	39.188
18.500	3.773159	0.146544	0.612469	0.205629	0.015995	0.005116	40.726
19.000	3.830180	0.147813	0.645434	0.219990	0.018142	0.004493	42.289
19.500	3.884915	0.148342	0.679081	0.234680	0.020494	0.003692	43.879
20.000	3.937464	0.148057	0.713474	0.249711	0.023066	0.002695	45.497
20.500	3.987097	0.146879	0.748695	0.265104	0.025874	0.001484	47.143
21.000	4.034345	0.144717	0.784851	0.280889	0.028936	0.000040	48.821
21.500	4.078508	0.141468	0.822070	0.297103	0.032274	-0.001658	50.531
22.000	4.119609	0.137016	0.860509	0.313792	0.035911	-0.003627	52.277
22.500	4.157217	0.131232	0.900349	0.331006	0.039873	-0.005885	54.062
23.000	4.190920	0.123980	0.941796	0.348794	0.044191	-0.008447	55.891
23.500	4.220217	0.115123	0.985077	0.367198	0.048890	-0.011318	57.767
24.000	4.244515	0.104545	1.030425	0.386239	0.053997	-0.014494	59.695
24.500	4.263144	0.092182	1.078057	0.405896	0.059526	-0.017955	61.680
25.000	4.275380	0.078069	1.128141	0.426081	0.065474	-0.021661	63.725
25.500	4.280478	0.062397	1.180770	0.446614	0.071809	-0.025553	65.831
25.550	4.280566	0.060758	1.186174	0.448676	0.072461	-0.025950	66.045
25.600	4.280573	0.059106	1.191604	0.450739	0.073117	-0.026348	66.259
26.343	4.270575	0.033610	1.275424	0.481195	0.083174	-0.032375	69.518
26.641	4.260573	0.023209	1.310761	0.493098	0.087319	-0.034850	70.860
26.859	4.250573	0.015668	1.337450	0.501628	0.090380	-0.036706	71.856
27.035	4.240574	0.009719	1.359503	0.508332	0.092848	-0.038241	72.667
27.183	4.230573	0.004853	1.378486	0.513818	0.094917	-0.039573	73.353
27.309	4.220573	0.000801	1.395211	0.518406	0.096689	-0.040763	73.946
27.419	4.210573	-0.002606	1.410168	0.522289	0.098225	-0.041847	74.465
27.514	4.200573	-0.005481	1.423684	0.525598	0.099567	-0.042849	74.924
27.599	4.190573	-0.007906	1.435988	0.528424	0.100745	-0.043785	75.332
27.673	4.180573	-0.009946	1.447253	0.530839	0.101782	-0.044669	75.694
27.738	4.170573	-0.011650	1.457613	0.532896	0.102695	-0.045509	76.018
27.796	4.160573	-0.013058	1.467173	0.534639	0.103498	-0.046313	76.303





Table II (cont.) - Values of deflection coefficients for various values of axial compressive load in the x-direction,  $P$ , for simply supported rectangular plate,  $a/b = 1.00$ ,  $\mu = 0.300$ .  
Normal pressure,  $p = 7.50 E h^4 / b^4$ .

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{1,3}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{3,3}}{h}$	$\frac{w_{1,5}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{eb^2}{h^2}$
27.846	4.150573	-0.014204	1.476021	0.536103	0.104203	-0.047085	76.563
27.890	4.140573	-0.015116	1.484228	0.537317	0.104821	-0.047830	76.791
27.928	4.130573	-0.015817	1.491858	0.538308	0.105358	-0.048551	76.993
27.960	4.120573	-0.016330	1.498961	0.539097	0.105823	-0.049252	77.171
27.988	4.110573	-0.016670	1.505584	0.539701	0.106222	-0.049934	77.328
28.011	4.100573	-0.016855	1.511767	0.540138	0.106560	-0.050600	77.464
28.030	4.090573	-0.016899	1.517544	0.540422	0.106842	-0.051251	77.582
28.045	4.080572	-0.016813	1.522947	0.540566	0.107073	-0.051888	77.683
28.057	4.070572	-0.016609	1.528004	0.540580	0.107257	-0.052513	77.767
28.065	4.060572	-0.016298	1.532740	0.540476	0.107397	-0.053127	77.837
28.070	4.050572	-0.015887	1.537178	0.540262	0.107497	-0.053730	77.892
28.072	4.040572	-0.015385	1.541339	0.539947	0.107559	-0.054322	77.935
28.071	4.030572	-0.014799	1.545240	0.539538	0.107586	-0.054905	77.965



Table II (cont.) - Values of reflection coefficients for various values of  $\alpha$  and constant  $\alpha = 0.263$ .

$\frac{F^2}{g}$	$\frac{\mu}{\gamma^2}$	$\frac{\mu}{\gamma^2}$	$\frac{\mu}{\gamma^2}$	$\frac{\mu}{\gamma^2}$	$\frac{\mu}{\gamma^2}$	$\frac{\mu}{\gamma^2}$	$\frac{\mu}{\gamma^2}$
38.034	4.084435	-0.0144309	1.247179	0.220438	0.104440	-0.024300	11.1652
38.035	4.083638	-0.014832	1.246181	0.220514	0.104320	-0.024333	11.032
38.040	4.082838	-0.014881	1.245183	0.240343	0.104404	-0.023840	11.453
38.042	4.082812	-0.014858	1.245100	0.240430	0.104331	-0.023135	11.421
38.043	4.082313	-0.014890	1.239064	0.240260	0.104321	-0.022813	11.484
38.044	4.081813	-0.0148612	1.239011	0.240200	0.104043	-0.021068	11.292
38.030	4.080813	-0.014860	1.211749	0.240000	0.103945	-0.021000	11.493
38.017	4.080313	-0.014822	1.211781	0.240000	0.103900	-0.020700	11.491
38.035	4.081633	-0.014819	1.209394	0.220301	0.103855	-0.020434	11.498
38.036	4.080312	-0.014832	1.209301	0.220301	0.104253	-0.043529	11.441
38.033	4.080943	-0.014771	1.211708	0.220230	0.102328	-0.043231	11.423
38.030	4.080013	-0.014718	1.209520	0.220314	0.104331	-0.043230	11.451
38.042	4.080313	-0.0147504	1.210031	0.220103	0.104303	-0.043102	11.419

Normal pressure,  $p = 1.50 \text{ kg/cm}^2$ .

the  $x$ -direction,  $\mu$ , for steady anisotropic hexagonal space,  $\alpha = 1.40$ ,  $\alpha = 0.263$ .

Table III

Values of deflection coefficients for various values of axial compressive load in the x-direction,  $P$ , for simply supported rectangular plate,  $a/b=1.00$ ,  $\mu=0.300$ .  
Normal pressure,  $p=12.50 E h^4/b^4$ .

$\frac{Pb^3}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{1,3}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{3,3}}{h}$	$\frac{w_{1,5}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{eb^2}{h^2}$
0.	0.521574	0.008252	0.008252	0.000856	0.000672	0.000672	0.336
0.500	0.583586	0.008596	0.008983	0.000897	0.000673	0.000687	0.921
1.000	0.656741	0.009091	0.009947	0.000951	0.000674	0.000702	1.533
1.500	0.741808	0.009806	0.011249	0.001029	0.000676	0.000720	2.180
2.000	0.838433	0.010816	0.013027	0.001143	0.000679	0.000740	2.869
2.500	0.944901	0.012197	0.015433	0.001312	0.000684	0.000764	3.604
3.000	1.058458	0.013999	0.018613	0.001562	0.000691	0.000795	4.386
3.500	1.176059	0.016239	0.022681	0.001923	0.000701	0.000835	5.212
4.000	1.295044	0.018902	0.027715	0.002429	0.000716	0.000886	6.078
4.500	1.413472	0.021955	0.033760	0.003116	0.000733	0.000951	6.978
5.000	1.530103	0.025354	0.040840	0.004021	0.000760	0.001032	7.907
28.582	4.108265	-0.025822	1.565221	0.557478	0.112708	-0.053915	80.164
28.589	4.098265	-0.025366	1.569745	0.557257	0.112808	-0.054531	80.226
28.592	4.088265	-0.024813	1.573976	0.556930	0.112869	-0.055139	80.274
28.592	4.078265	-0.024172	1.577937	0.556504	0.112893	-0.055739	80.308
28.590	4.068265	-0.023451	1.581643	0.555988	0.112882	-0.056330	80.351





Table IV - Values of deflection coefficients for various values of axial compressive load in the x-direction,  $P$ , for simply supported rectangular plate,  $a/b = 1.00$ ,  $\nu = 0.300$ .  
Normal pressure,  $p = 18.00 E h^4 / b^4$ .

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{1,3}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{3,5}}{h}$	$\frac{w_{1,5}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{e_b^2}{h^2}$
0.	0.704259	0.012496	0.012496	0.001334	0.000938	0.000968	0.613
0.500	0.773978	0.013128	0.013706	0.001419	0.000970	0.000990	1.241
1.000	0.852085	0.013965	0.015241	0.001533	0.000972	0.001014	1.898
1.500	0.938231	0.015057	0.017200	0.001688	0.000976	0.001041	2.589
2.000	1.031473	0.016453	0.019688	0.001900	0.000981	0.001072	3.317
2.500	1.130381	0.018189	0.022809	0.002189	0.000989	0.001109	4.082
3.000	1.233292	0.020282	0.026658	0.002580	0.000999	0.001153	4.884
3.500	1.338575	0.022732	0.031311	0.003101	0.001013	0.001207	5.722
4.000	1.444828	0.025526	0.036825	0.003782	0.001031	0.001273	6.591
4.500	1.550963	0.028639	0.043241	0.004654	0.001053	0.001354	7.489
5.000	1.656205	0.032044	0.050589	0.005750	0.001080	0.001450	8.414
29.156	4.159187	-0.036514	1.600875	0.575991	0.118569	-0.054616	82.749
29.164	4.149187	-0.036003	1.605478	0.575758	0.118669	-0.055247	82.817
29.169	4.139187	-0.035390	1.609775	0.575413	0.118726	-0.055871	82.870
29.170	4.129187	-0.034684	1.613785	0.574966	0.118744	-0.056488	82.910
29.169	4.119187	-0.033893	1.617529	0.574424	0.118726	-0.057098	82.936



Table V- Values of deflection coefficients for various values of axial compressive load in the x-direction,  $P$ , for simply supported rectangular plate,  $a/b=1.00$ ,  $\mu=0.300$ .  
Normal pressure,  $p=24.50 \text{ Eh}^4/b^4$ .

$\frac{Pb^3}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{1,3}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{3,5}}{h}$	$\frac{w_{1,5}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{eb^2}{h^2}$
0.	0.887324	0.017832	0.017832	0.002001	0.001318	0.001318	0.975
0.500	0.959922	0.018778	0.019579	0.002157	0.001322	0.001349	1.641
1.000	1.038418	0.019949	0.021710	0.002359	0.001326	0.001383	2.336
1.500	1.122226	0.021376	0.024302	0.002619	0.001332	0.001421	3.060
2.000	1.210522	0.023084	0.027434	0.002956	0.001340	0.001465	3.817
2.500	1.302331	0.025090	0.031179	0.003389	0.001350	0.001516	4.604
3.000	1.396643	0.027398	0.035603	0.003942	0.001364	0.001575	5.421
3.500	1.492512	0.030004	0.040762	0.004640	0.001381	0.001645	6.268
4.000	1.589121	0.032895	0.046704	0.005510	0.001401	0.001727	7.141
4.500	1.685807	0.036054	0.053465	0.006581	0.001426	0.001822	8.040
5.000	1.782061	0.039465	0.061077	0.007881	0.001456	0.001934	8.962
29.845	4.216877	-0.046362	1.643312	0.597627	0.125536	-0.055255	85.849
29.854	4.206877	-0.048771	1.647968	0.597366	0.125629	-0.055903	85.923
29.860	4.196877	-0.048674	1.652300	0.596987	0.125677	-0.056545	85.931
29.863	4.186877	-0.047275	1.656320	0.596502	0.125683	-0.057182	86.024
29.861	4.176877	-0.046396	1.660081	0.595517	0.125651	-0.057014	86.052



20' 591	4' 14611	-0' 046308	1' 660221	0' 289211	0' 728021	-0' 064217	10' 022
20' 603	4' 14611	-0' 046312	1' 660220	0' 289208	0' 728013	-0' 064188	10' 024
20' 615	4' 14611	-0' 046314	1' 660219	0' 289205	0' 728011	-0' 064142	10' 026
20' 627	4' 14611	-0' 046316	1' 660218	0' 289202	0' 728009	-0' 064096	10' 028
20' 639	4' 14611	-0' 046318	1' 660217	0' 289199	0' 728007	-0' 064050	10' 030
20' 651	4' 14611	-0' 046320	1' 660216	0' 289196	0' 728005	-0' 064004	10' 032
20' 663	4' 14611	-0' 046322	1' 660215	0' 289193	0' 728003	-0' 063958	10' 034
20' 675	4' 14611	-0' 046324	1' 660214	0' 289190	0' 728001	-0' 063912	10' 036
20' 687	4' 14611	-0' 046326	1' 660213	0' 289187	0' 728000	-0' 063866	10' 038
20' 699	4' 14611	-0' 046328	1' 660212	0' 289184	0' 728000	-0' 063820	10' 040

2' 090	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 628
2' 100	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 640
2' 110	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 652
2' 120	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 664
2' 130	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 676
2' 140	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 688
2' 150	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 700
2' 160	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 712
2' 170	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 724
2' 180	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 736
2' 190	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 748
2' 200	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 760
2' 210	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 772
2' 220	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 784
2' 230	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 796
2' 240	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 808
2' 250	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 820
2' 260	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 832
2' 270	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 844
2' 280	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 856
2' 290	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 868
2' 300	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 880
2' 310	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 892
2' 320	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 904
2' 330	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 916
2' 340	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 928
2' 350	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 940
2' 360	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 952
2' 370	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 964
2' 380	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 976
2' 390	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 988
2' 400	1' 123061	0' 023709	0' 041711	0' 001001	0' 001400	0' 001034	8' 100

$\frac{2\pi^2}{3}$	$\frac{1}{\pi}$	$\frac{1}{\pi^2}$	$\frac{1}{\pi^3}$	$\frac{1}{\pi^4}$	$\frac{1}{\pi^5}$	$\frac{1}{\pi^6}$	$\frac{1}{\pi^7}$
0.628318	0.318309	0.031416	0.003141	0.000314	0.000031	0.000003	0.000000

Normal measure,  $\mu = 84' 20''$   $\mu^2 = 10' 44''$   
the 2- $\pi$  angle,  $\mu^2$  for simply connected regions,  $\mu^2 = 10' 44''$   $\mu^2 = 10' 44''$

At the  $\mu^2$  -  $\mu^2$  of collection coefficients for various angles of  $\mu^2$  coefficients  $\mu^2$  to

Table VI - Values of deflection coefficients for various values of axial compressive load in the x-direction,  $P$ , for simply supported rectangular plate,  $a/b = 1.00$ ,  $\mu = 0.300$ .  
Normal pressure,  $p = 30.00 \text{ } E h^4 / b^4$ .

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{1,3}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{3,3}}{h}$	$\frac{w_{1,5}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{e_b^2}{h^2}$
0.	1.020802	0.022490	0.022490	0.002652	0.001615	0.001615	1.291
0.500	1.093572	0.023659	0.024643	0.002878	0.001620	0.001654	1.983
1.000	1.170845	0.025052	0.027203	0.003161	0.001626	0.001696	2.700
1.500	1.252071	0.026690	0.030234	0.003516	0.001633	0.001744	3.445
2.000	1.336590	0.028587	0.033798	0.003961	0.001643	0.001798	4.217
2.500	1.423682	0.030751	0.037955	0.004514	0.001656	0.001859	5.018
3.000	1.512635	0.033183	0.042753	0.005199	0.001671	0.001930	5.844
3.500	1.602790	0.035877	0.048255	0.006038	0.001690	0.002011	6.697
4.000	1.693571	0.038824	0.054486	0.007057	0.001713	0.002105	7.574
4.500	1.784501	0.042011	0.061486	0.008282	0.001740	0.002213	8.473
5.000	1.875199	0.045424	0.069285	0.009742	0.001772	0.002336	9.395
30.436	4.263910	-0.060373	1.679468	0.615745	0.131466	-0.055621	88.516
30.446	4.253910	-0.059704	1.684135	0.615448	0.131348	-0.056283	88.593
30.453	4.243909	-0.058926	1.688463	0.615031	0.131583	-0.056940	88.653
30.455	4.233909	-0.058047	1.692479	0.614503	0.131574	-0.057594	88.697
30.454	4.223909	-0.057077	1.696205	0.613874	0.131526	-0.058245	88.725

[illegible][illegible][illegible]

Machung Hirschen: d = 20.00 €

$$0.01 \pm 0.001 = 0.01 \pm 0.001$$



Table VII - Values of deflection coefficients for various values of axial compressive load in the x-direction,  $P$ , for simply supported rectangular plate,  $a/b = 4.00$ ,  $\mu = 0.300$ . Normal pressure,  $p = 2.50 Eh^4/b^4$ .

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{w_{7,1}}{h}$	$\frac{eb^2}{h^2}$
0.	0.383	0.068	0.015	0.004	0.015
0.500	0.395	0.077	0.018	0.005	0.516
1.000	0.407	0.088	0.021	0.005	1.019
1.500	0.421	0.103	0.026	0.007	1.522
2.000	0.436	0.123	0.034	0.008	2.027
2.500	0.454	0.150	0.045	0.010	2.535
3.000	0.473	0.185	0.064	0.014	3.049
3.500	0.493	0.230	0.099	0.019	3.575
4.000	0.508	0.263	0.188	0.020	4.137
4.101	0.498	0.256	0.238	0.015	4.276
4.133	0.488	0.249	0.264	0.011	4.329
4.151	0.478	0.243	0.285	0.008	4.366
4.163	0.468	0.237	0.303	0.006	4.396
4.172	0.458	0.232	0.319	0.003	4.422
4.179	0.448	0.227	0.335	0.001	4.446
4.185	0.438	0.221	0.349	-0.001	4.469
4.190	0.428	0.216	0.364	-0.003	4.492
4.195	0.418	0.212	0.378	-0.005	4.514
4.199	0.408	0.207	0.391	-0.006	4.537
4.204	0.398	0.202	0.405	-0.008	4.561
4.209	0.388	0.197	0.419	-0.009	4.586
4.215	0.378	0.192	0.432	-0.011	4.612
4.221	0.368	0.188	0.446	-0.012	4.640
4.228	0.358	0.183	0.460	-0.013	4.670
4.236	0.348	0.178	0.474	-0.015	4.701
4.245	0.338	0.174	0.488	-0.016	4.736
4.256	0.328	0.169	0.503	-0.017	4.772
4.267	0.318	0.164	0.518	-0.018	4.812
4.280	0.308	0.160	0.533	-0.019	4.854
4.294	0.298	0.155	0.549	-0.019	4.901
4.310	0.288	0.150	0.566	-0.020	4.951
4.329	0.278	0.146	0.583	-0.021	5.006
4.349	0.268	0.141	0.601	-0.021	5.065
4.372	0.258	0.136	0.619	-0.022	5.130
4.397	0.248	0.131	0.638	-0.023	5.202
4.426	0.238	0.127	0.659	-0.023	5.280
4.458	0.228	0.122	0.680	-0.023	5.366
4.494	0.218	0.117	0.703	-0.024	5.461
4.534	0.208	0.113	0.727	-0.024	5.567

Table VII - Values of deflection coefficients for various values of  $\alpha$  and  $\beta$  for rectangular plate,  $\alpha/\beta = 1.00, 1.50, 2.00, 2.50, 3.00, 3.50, 4.00, 4.50, 5.00, 5.50, 6.00, 6.50, 7.00, 7.50, 8.00, 8.50, 9.00, 9.50, 10.00$

$\frac{w}{b}$	$\frac{w}{b}$	$\frac{w}{b}$	$\frac{w}{b}$	$\frac{w}{b}$	$\frac{w}{b}$
0.000	0.000	0.000	0.000	0.000	0.000
0.001	0.001	0.001	0.001	0.001	0.001
0.002	0.002	0.002	0.002	0.002	0.002
0.003	0.003	0.003	0.003	0.003	0.003
0.004	0.004	0.004	0.004	0.004	0.004
0.005	0.005	0.005	0.005	0.005	0.005
0.006	0.006	0.006	0.006	0.006	0.006
0.007	0.007	0.007	0.007	0.007	0.007
0.008	0.008	0.008	0.008	0.008	0.008
0.009	0.009	0.009	0.009	0.009	0.009
0.010	0.010	0.010	0.010	0.010	0.010
0.011	0.011	0.011	0.011	0.011	0.011
0.012	0.012	0.012	0.012	0.012	0.012
0.013	0.013	0.013	0.013	0.013	0.013
0.014	0.014	0.014	0.014	0.014	0.014
0.015	0.015	0.015	0.015	0.015	0.015
0.016	0.016	0.016	0.016	0.016	0.016
0.017	0.017	0.017	0.017	0.017	0.017
0.018	0.018	0.018	0.018	0.018	0.018
0.019	0.019	0.019	0.019	0.019	0.019
0.020	0.020	0.020	0.020	0.020	0.020
0.021	0.021	0.021	0.021	0.021	0.021
0.022	0.022	0.022	0.022	0.022	0.022
0.023	0.023	0.023	0.023	0.023	0.023
0.024	0.024	0.024	0.024	0.024	0.024
0.025	0.025	0.025	0.025	0.025	0.025
0.026	0.026	0.026	0.026	0.026	0.026
0.027	0.027	0.027	0.027	0.027	0.027
0.028	0.028	0.028	0.028	0.028	0.028
0.029	0.029	0.029	0.029	0.029	0.029
0.030	0.030	0.030	0.030	0.030	0.030
0.031	0.031	0.031	0.031	0.031	0.031
0.032	0.032	0.032	0.032	0.032	0.032
0.033	0.033	0.033	0.033	0.033	0.033
0.034	0.034	0.034	0.034	0.034	0.034
0.035	0.035	0.035	0.035	0.035	0.035
0.036	0.036	0.036	0.036	0.036	0.036
0.037	0.037	0.037	0.037	0.037	0.037
0.038	0.038	0.038	0.038	0.038	0.038
0.039	0.039	0.039	0.039	0.039	0.039
0.040	0.040	0.040	0.040	0.040	0.040
0.041	0.041	0.041	0.041	0.041	0.041
0.042	0.042	0.042	0.042	0.042	0.042
0.043	0.043	0.043	0.043	0.043	0.043
0.044	0.044	0.044	0.044	0.044	0.044
0.045	0.045	0.045	0.045	0.045	0.045
0.046	0.046	0.046	0.046	0.046	0.046
0.047	0.047	0.047	0.047	0.047	0.047
0.048	0.048	0.048	0.048	0.048	0.048
0.049	0.049	0.049	0.049	0.049	0.049
0.050	0.050	0.050	0.050	0.050	0.050

Table VIII - Values of deflection coefficients for various values of axial compressive load in the x-direction,  $P$ , for simply supported rectangular plate,  $a/b = 4.00$ ,  $\mu = 0.300$ . Normal pressure,  $p = 7.50 Eh^4/b^4$ .

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{w_{7,1}}{h}$	$\frac{eb^2}{h^2}$
0.	0.961	0.223	0.064	0.018	0.115
0.500	0.986	0.242	0.073	0.021	0.627
1.000	1.014	0.264	0.085	0.024	1.143
1.500	1.044	0.288	0.099	0.029	1.663
2.000	1.076	0.314	0.116	0.035	2.188
2.500	1.112	0.342	0.137	0.043	2.719
3.000	1.151	0.371	0.162	0.053	3.258
3.500	1.193	0.401	0.190	0.066	3.807
4.000	1.237	0.430	0.223	0.081	4.366
4.500	1.282	0.457	0.259	0.100	4.938
5.000	1.328	0.482	0.298	0.122	5.524
6.727	1.416	0.499	0.469	0.250	7.716
6.780	1.406	0.491	0.482	0.257	7.796
6.805	1.396	0.484	0.492	0.259	7.838
6.815	1.386	0.477	0.501	0.259	7.857
6.815	1.376	0.471	0.509	0.256	7.863



$\frac{d}{dt} \ln C_0 = -\lambda$ [illegible]

Table IX - Values of deflection coefficients for various values of axial compressive load in the x-direction,  $P$ , for simply supported rectangular plate,  $a/b = 4.00$ ,  $\mu = 0.300$ .  
Normal pressure,  $p = 12.50 Eh^4/b^4$ .

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{w_{7,1}}{h}$	$\frac{eb^2}{h^2}$
0.	1.373	.352	.118	.037	.263
.500	1.404	.374	.132	.042	.789
1.000	1.437	.396	.147	.049	1.318
1.500	1.471	.420	.163	.056	1.852
2.000	1.508	.445	.182	.065	2.392
2.500	1.546	.470	.202	.075	2.937
3.000	1.587	.495	.225	.087	3.490
3.500	1.629	.521	.248	.100	4.050
4.000	1.672	.546	.274	.116	4.617
4.500	1.717	.571	.300	.134	5.194
5.00	1.761	.595	.327	.154	5.779
8.669	1.900	.659	.488	.491	10.617
8.675	1.890	.655	.486	.498	10.640
8.677	1.880	.651	.484	.504	10.658
8.677	1.870	.648	.483	.511	10.672
8.675	1.860	.644	.481	.517	10.684

Teste IX - Values of deflection coefficients for various values of axial compressive load in the direction,  $P$ , for a supported rectangular plate,  $a/b = 4.00$ ,  $\mu = 0.100$ .  
 Normal pressure,  $p = 12.50 \text{ lb./in.}^2$ .

$\frac{P_0}{3}$	$\frac{W_{1,1}}{b}$	$\frac{W_{2,1}}{a}$	$\frac{W_{3,1}}{b}$	$\frac{W_{4,1}}{a}$	$\frac{E\delta}{a}$
0.	1.373	.723	.118	.027	1.203
.500	1.404	.774	.123	.042	.753
1.000	1.437	.826	.127	.058	1.218
1.500	1.471	.880	.132	.075	1.622
2.000	1.504	.934	.137	.092	2.028
2.500	1.546	.990	.142	.117	2.437
3.000	1.587	.1048	.147	.142	2.847
3.500	1.628	.1071	.152	.167	3.250
4.000	1.672	.1094	.157	.192	3.657
4.500	1.717	.1117	.162	.217	4.064
5.00	1.761	.1140	.167	.242	4.470

5.500	1.806	.1163	.172	.267	4.877
6.000	1.850	.1186	.177	.292	5.283
6.500	1.894	.1209	.182	.317	5.689
7.000	1.938	.1232	.187	.342	6.096
7.500	1.982	.1255	.192	.367	6.502
8.000	2.026	.1278	.197	.392	6.908
8.500	2.070	.1301	.202	.417	7.314
9.000	2.114	.1324	.207	.442	7.720
9.500	2.158	.1347	.212	.467	8.126
10.000	2.202	.1370	.217	.492	8.532



Table X - Values of deflection coefficients for various values of axial compressive load in the x-direction,  $P$ , for simply supported rectangular plate,  $a/b = 4.00$ ,  $\mu = 0.300$ . Normal pressure,  $p = 18.00 Eh^4/b^4$ .

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{w_{7,1}}{h}$	$\frac{e_{b^2}}{h^2}$
01.	1.723	0.464	0.171	0.059	0.448
0.500	1.755	0.485	0.186	0.066	0.984
1.000	1.789	0.507	0.202	0.074	1.524
1.500	1.824	0.529	0.219	0.082	2.068
2.000	1.861	0.552	0.237	0.092	2.618
2.500	1.899	0.575	0.256	0.103	3.173
3.000	1.938	0.597	0.276	0.115	3.733
3.500	1.978	0.620	0.296	0.129	4.300
4.000	2.018	0.643	0.318	0.144	4.873
4.500	2.059	0.665	0.340	0.160	5.452
5.000	2.101	0.687	0.362	0.178	6.039
10.247	2.305	0.795	0.544	0.578	12.930
10.254	2.295	0.792	0.543	0.585	12.956
10.258	2.285	0.789	0.541	0.591	12.977
10.260	2.275	0.785	0.539	0.597	12.995
10.258	2.265	0.782	0.536	0.603	13.009

$\sigma = 11.0 \pm 0.7 \text{ eV}$   
 $\sigma = 4.00, \sigma = 0.100$ . Values in parentheses are  
 compressive load in the direction of the slowly supported  
 Values of reflection coefficients for various values of angle

$\frac{v}{v_0}$	$\frac{1}{v}$	$\frac{1}{v^2}$	$\frac{1}{v^3}$	$\frac{1}{v^4}$	$\frac{1}{v^5}$
0.000	0.000	0.000	0.000	0.000	0.000
0.001	0.001	0.001	0.001	0.001	0.001
0.002	0.002	0.004	0.008	0.016	0.032
0.003	0.003	0.009	0.027	0.081	0.243
0.004	0.004	0.016	0.064	0.256	1.024
0.005	0.005	0.025	0.125	0.625	3.125
0.006	0.006	0.036	0.216	1.296	7.776
0.007	0.007	0.049	0.343	2.401	16.807
0.008	0.008	0.064	0.512	4.096	32.768
0.009	0.009	0.081	0.729	6.561	59.049
0.010	0.010	0.100	1.000	10.000	100.000
0.011	0.011	0.121	1.331	14.641	161.051
0.012	0.012	0.144	1.728	20.736	248.832
0.013	0.013	0.169	2.197	28.549	371.233
0.014	0.014	0.196	2.744	38.264	537.824
0.015	0.015	0.225	3.375	50.625	759.375
0.016	0.016	0.256	4.096	65.536	1048.576
0.017	0.017	0.289	4.913	83.521	1419.857
0.018	0.018	0.324	5.832	104.976	1889.592
0.019	0.019	0.361	6.859	130.309	2519.119
0.020	0.020	0.400	8.000	160.000	3200.000
0.021	0.021	0.441	9.261	194.481	4096.481
0.022	0.022	0.484	10.648	234.256	5196.928
0.023	0.023	0.529	12.167	280.089	6585.007
0.024	0.024	0.576	13.824	332.944	8294.400
0.025	0.025	0.625	15.625	394.531	10375.000
0.026	0.026	0.676	17.576	464.876	12984.656
0.027	0.027	0.729	19.683	544.081	16200.000
0.028	0.028	0.784	21.952	632.672	20128.000
0.029	0.029	0.841	24.389	731.269	24896.000
0.030	0.030	0.900	27.000	842.400	30600.000
0.031	0.031	0.961	29.791	967.641	37504.000
0.032	0.032	1.024	32.768	1108.224	45776.000
0.033	0.033	1.089	35.937	1265.841	55568.000
0.034	0.034	1.156	39.304	1441.216	67056.000
0.035	0.035	1.225	42.875	1635.063	80400.000
0.036	0.036	1.296	46.656	1848.064	95776.000
0.037	0.037	1.369	50.653	2081.009	113360.000
0.038	0.038	1.444	54.872	2335.632	133328.000
0.039	0.039	1.521	59.313	2613.681	155856.000
0.040	0.040	1.600	64.000	2916.000	180800.000
0.041	0.041	1.681	68.921	3244.881	208336.000
0.042	0.042	1.764	74.088	3601.944	238640.000
0.043	0.043	1.849	79.503	3988.849	271904.000
0.044	0.044	1.936	85.176	4407.376	308320.000
0.045	0.045	2.025	91.125	4859.405	348000.000
0.046	0.046	2.116	97.336	5346.976	391136.000
0.047	0.047	2.209	103.803	5871.949	437920.000
0.048	0.048	2.304	110.528	6436.224	488544.000
0.049	0.049	2.401	117.513	7041.841	543200.000
0.050	0.050	2.500	124.750	7687.500	602000.000
0.051	0.051	2.601	132.251	8374.851	665241.000
0.052	0.052	2.704	140.000	9104.000	733216.000
0.053	0.053	2.809	147.993	9876.081	806129.000
0.054	0.054	2.916	156.240	10692.816	884280.000
0.055	0.055	3.025	164.750	11556.125	967880.000</

Table XI - Values of deflection coefficients for various values of axial compressive load in the x-direction,  $P$ , for simply supported rectangular plate,  $a/b=4.00$ ,  $\mu = 0.300$ . Normal pressure,  $p = 24.50 Eh^4/b^4$ .

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{w_{7,1}}{h}$	$\frac{eb^2}{h^2}$
0.	2.053	0.569	0.224	0.083	0.672
0.500	2.085	0.589	0.238	0.091	1.216
1.000	2.118	0.610	0.254	0.099	1.765
1.500	2.153	0.630	0.270	0.109	2.318
2.000	2.188	0.651	0.287	0.119	2.875
2.500	2.224	0.671	0.304	0.130	3.433
3.000	2.261	0.692	0.322	0.142	4.003
3.500	2.298	0.713	0.341	0.155	4.574
4.000	2.336	0.733	0.360	0.169	5.150
4.500	2.374	0.753	0.379	0.184	5.732
5.000	2.412	0.773	0.398	0.199	6.319
11.964	2.675	0.916	0.607	0.659	15.448
11.973	2.665	0.913	0.605	0.665	15.478
11.978	2.655	0.910	0.603	0.672	15.503
11.981	2.645	0.907	0.601	0.678	15.523
11.981	2.635	0.904	0.599	0.683	15.540



Table XI  
Values of reflection coefficients for various values of axial compressive load in the x-direction,  $E$ , for simply supported rectangular plate,  $a/b=1.00$ ,  $\nu=0.300$ . Horizontal pressure,  $p=24.50 \text{ lb/in}^2$ .

$\frac{p_0}{Eh}$	$\frac{w_{1,1}}{h}$	$\frac{w_{2,1}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{4,1}}{h}$	$\frac{w_{5,1}}{h}$
0.000	0.053	0.069	0.075	0.080	0.083
0.200	0.081	0.100	0.108	0.113	0.116
1.000	0.113	0.130	0.140	0.146	0.150
1.100	0.108	0.125	0.135	0.141	0.144
2.000	0.188	0.207	0.217	0.223	0.226
3.000	0.228	0.247	0.257	0.263	0.266
4.000	0.261	0.280	0.290	0.296	0.299
5.000	0.298	0.317	0.327	0.333	0.336
6.000	0.330	0.349	0.359	0.365	0.368
7.000	0.357	0.376	0.386	0.392	0.395
8.000	0.382	0.401	0.411	0.417	0.420
9.000	0.406	0.425	0.435	0.441	0.444
10.000	0.429	0.448	0.458	0.464	0.467
11.000	0.452	0.471	0.481	0.487	0.490
12.000	0.475	0.494	0.504	0.510	0.513
13.000	0.498	0.517	0.527	0.533	0.536
14.000	0.521	0.540	0.550	0.556	0.559
15.000	0.544	0.563	0.573	0.579	0.582
16.000	0.567	0.586	0.596	0.602	0.605
17.000	0.590	0.609	0.619	0.625	0.628
18.000	0.613	0.632	0.642	0.648	0.651
19.000	0.636	0.655	0.665	0.671	0.674
20.000	0.659	0.678	0.688	0.694	0.697
21.000	0.682	0.701	0.711	0.717	0.720
22.000	0.705	0.724	0.734	0.740	0.743
23.000	0.728	0.747	0.757	0.763	0.766
24.000	0.751	0.770	0.780	0.786	0.789
25.000	0.774	0.793	0.803	0.809	0.812
26.000	0.797	0.816	0.826	0.832	0.835
27.000	0.820	0.839	0.849	0.855	0.858
28.000	0.843	0.862	0.872	0.878	0.881
29.000	0.866	0.885	0.895	0.901	0.904
30.000	0.889	0.908	0.918	0.924	0.927
31.000	0.912	0.931	0.941	0.947	0.950
32.000	0.935	0.954	0.964	0.970	0.973
33.000	0.958	0.977	0.987	0.993	0.996
34.000	0.981	1.000	1.010	1.016	1.019
35.000	1.004	1.023	1.033	1.039	1.042
36.000	1.027	1.046	1.056	1.062	1.065
37.000	1.050	1.069	1.079	1.085	1.088
38.000	1.073	1.092	1.102	1.108	1.111
39.000	1.096	1.115	1.125	1.131	1.134
40.000	1.119	1.138	1.148	1.154	1.157
41.000	1.142	1.161	1.171	1.177	1.180
42.000	1.165	1.184	1.194	1.200	1.203
43.000	1.188	1.207	1.217	1.223	1.226
44.000	1.211	1.230	1.240	1.246	1.249
45.000	1.234	1.253	1.263	1.269	1.272
46.000	1.257	1.276	1.286	1.292	1.295
47.000	1.280	1.299	1.309	1.315	1.318
48.000	1.303	1.322	1.332	1.338	1.341
49.000	1.326	1.345	1.355	1.361	1.364
50.000	1.349	1.368	1.378	1.384	1.387
51.000	1.372	1.391	1.401	1.407	1.410
52.000	1.395	1.414	1.424	1.430	1.433
53.000	1.418	1.437	1.447	1.453	1.456
54.000	1.441	1.460	1.470	1.476	1.479
55.000	1.464	1.483	1.493	1.499	1.502
56.000	1.487	1.506	1.516	1.522	1.525
57.000	1.510	1.529	1.539	1.545	1.548
58.000	1.533	1.552	1.562	1.568	1.571
59.000	1.556	1.575	1.585	1.591	1.594
60.000	1.579	1.598	1.608	1.614	1.617
61.000	1.602	1.621	1.631	1.637	1.640
62.000	1.625	1.644	1.654	1.660	1.663
63.000	1.648	1.667	1.677	1.683	1.686
64.000	1.671	1.690	1.700	1.706	1.709
65.000	1.694	1.713	1.723	1.729	1.732
66.000	1.717	1.736	1.746	1.752	1.755
67.000	1.740	1.759	1.769	1.775	1.778
68.000	1.763	1.782	1.792	1.798	1.801
69.000	1.786	1.805	1.815	1.821	1.824
70.000	1.809	1.828	1.838	1.844	1.847
71.000	1.832	1.851	1.861	1.867	1.870
72.000	1.855	1.874	1.884	1.890	1.893
73.000	1.878	1.897	1.907	1.913	1.916
74.000	1.901	1.920	1.930	1.936	1.939
75.000	1.924	1.943	1.953	1.959	1.962
76.000	1.947	1.966	1.976	1.982	1.985
77.000	1.970	1.989	1.999	2.005	2.008
78.000	1.993	2.012	2.022	2.028	2.031
79.000	2.016	2.035	2.045	2.051	2.054
80.000	2.039	2.058	2.068	2.074	2.077
81.000	2.062	2.081	2.091	2.097	2.100
82.000	2.085	2.104	2.114	2.120	2.123
83.000	2.108	2.127	2.137	2.143	2.146
84.000	2.131	2.150	2.160	2.166	2.169
85.000	2.154	2.173	2.183	2.189	2.192
86.000	2.177	2.196	2.206	2.212	2.215
87.000	2.200	2.219	2.229	2.235	2.238
88.000	2.223	2.242	2.252	2.258	2.261
89.000	2.246	2.265	2.275	2.281	2.284
90.000	2.269	2.288	2.298	2.304	2.307
91.000	2.292	2.311	2.321	2.327	2.330
92.000	2.315	2.334	2.344	2.350	2.353
93.000	2.338	2.357	2.367	2.373	2.376
94.000	2.361	2.380	2.390	2.396	2.399
95.000	2.384	2.403	2.413	2.419	2.422
96.000	2.407	2.426	2.436	2.442	2.445
97.000	2.430	2.449	2.459	2.465	2.468
98.000	2.453	2.472	2.482	2.488	2.491
99.000	2.476	2.495	2.505	2.511	2.514
100.000	2.499	2.518	2.528	2.534	2.537

Table XII - Values of deflection coefficients for various values of axial compressive load in the x-direction,  $P$ , for simply supported rectangular plate,  $a/b = 4.00$ ,  $\mu = 0.300$ . Normal pressure,  $p = 30.00 Eh^4/b^4$ .

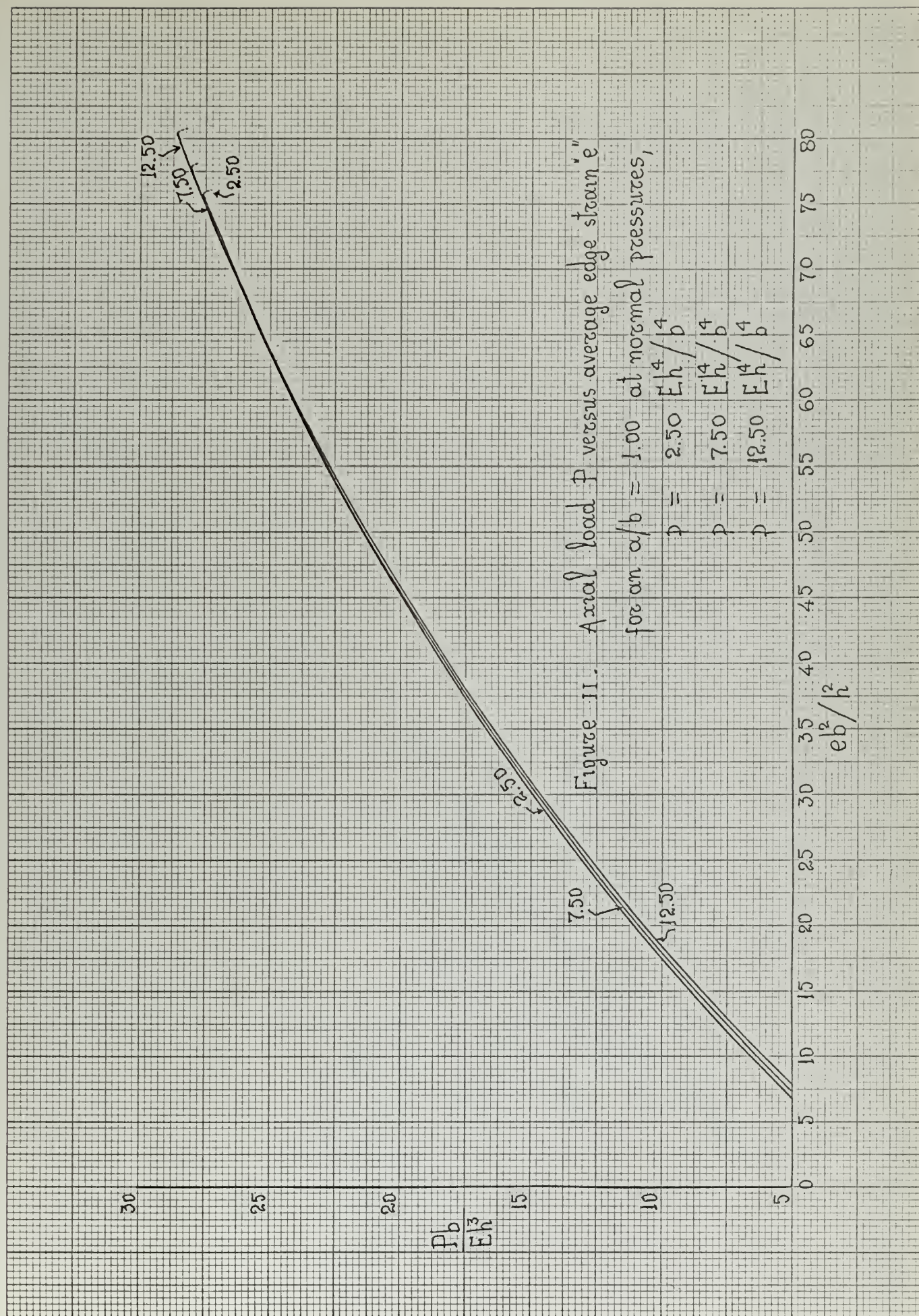
$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{w_{7,1}}{h}$	$\frac{e_b^2}{h^2}$
0.	2.286	0.643	0.261	0.101	0.860
0.500	2.318	0.662	0.275	0.110	1.409
1.000	2.350	0.681	0.290	0.118	1.963
1.500	2.384	0.701	0.306	0.128	2.520
2.000	2.418	0.720	0.322	0.138	3.082
2.500	2.452	0.739	0.338	0.149	3.647
3.000	2.487	0.759	0.355	0.161	4.217
3.500	2.522	0.778	0.372	0.173	4.791
4.000	2.558	0.797	0.390	0.186	5.369
4.500	2.594	0.816	0.407	0.200	5.952
5.000	2.630	0.835	0.425	0.215	6.539
13.331	2.920	0.996	0.651	0.724	17.476
13.337	2.910	0.993	0.649	0.730	17.502
13.339	2.900	0.990	0.648	0.736	17.524
13.339	2.890	0.987	0.646	0.742	17.542
13.337	2.880	0.984	0.643	0.747	17.556

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$p = 30.00 \text{ MN/m}^2$   
rectangular plate,  $a/b = 4.00$ ,  $\nu = 0.300$ . Normal pressure,  
compressive load in the  $x$ -direction,  $P$ , for simply supported  
Values of deflection coefficients for various values of axial

[illegible]









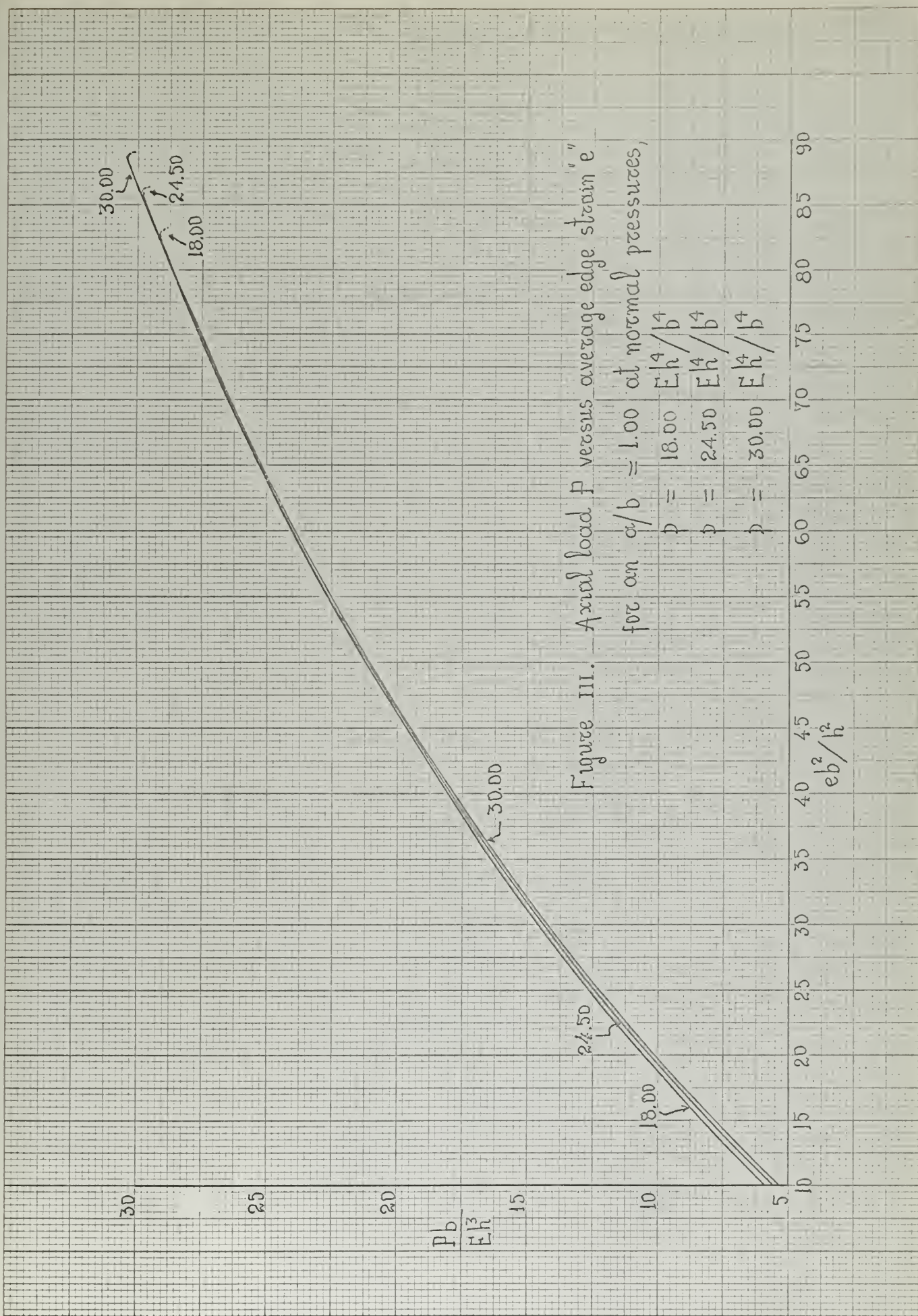
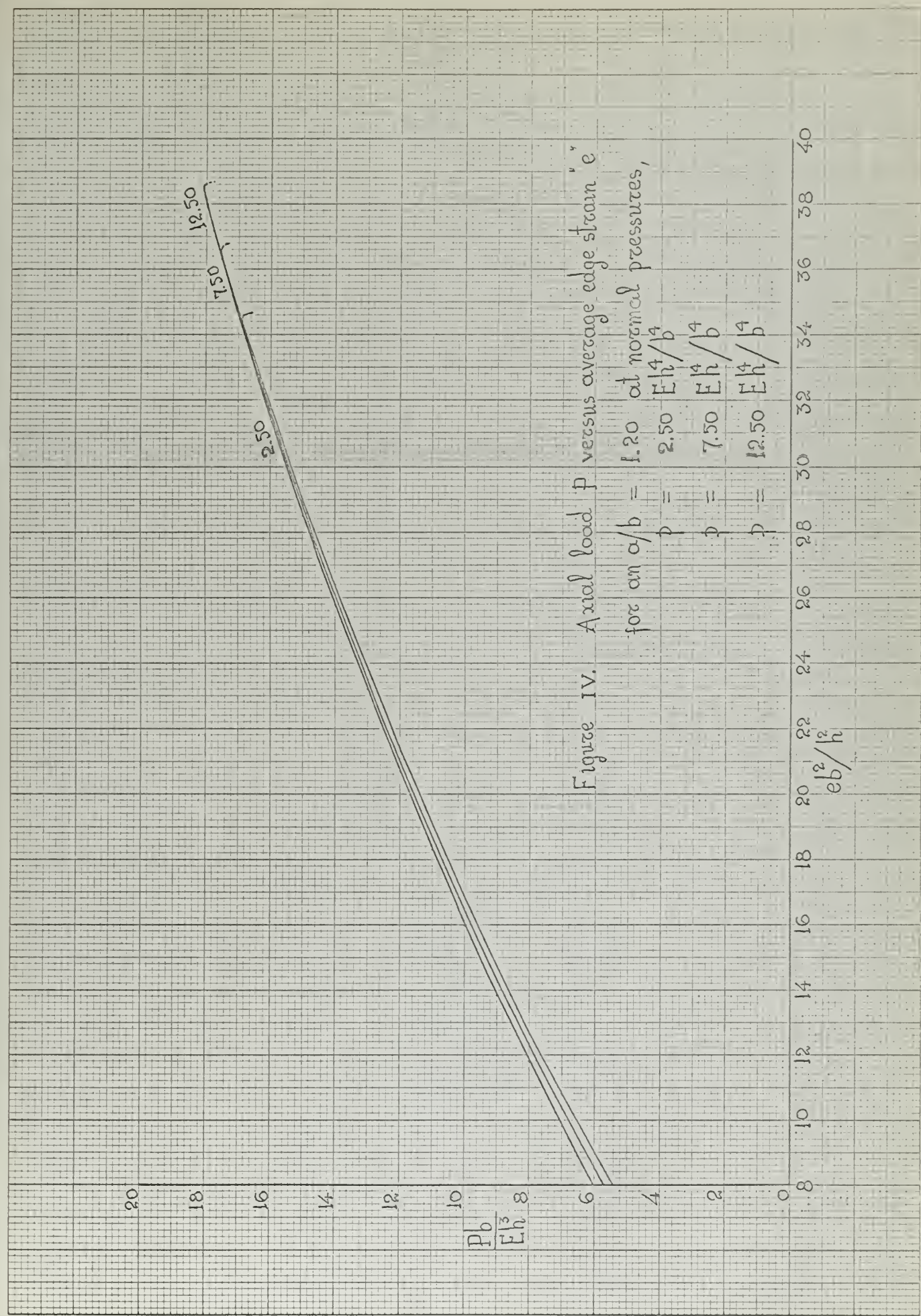


Figure III. Axial load  $P$  versus average edge strain  $e$   
for an  $a/b = 1.00$  at normal pressures,

$p = 18.00 \quad \frac{Eh^4}{b^4}$   
 $p = 24.50 \quad \frac{Eh^4}{b^4}$   
 $p = 30.00 \quad \frac{Eh^4}{b^4}$



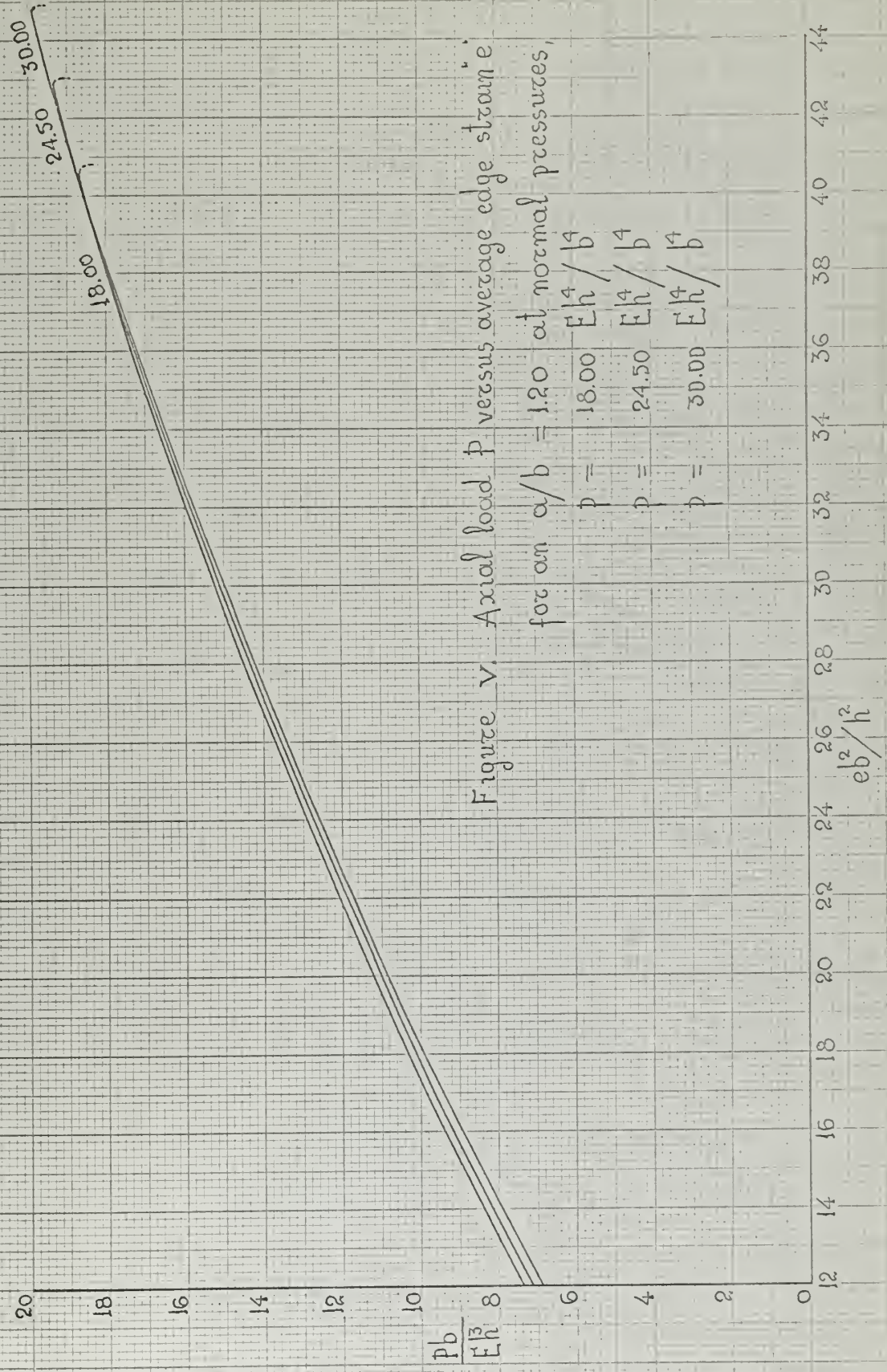






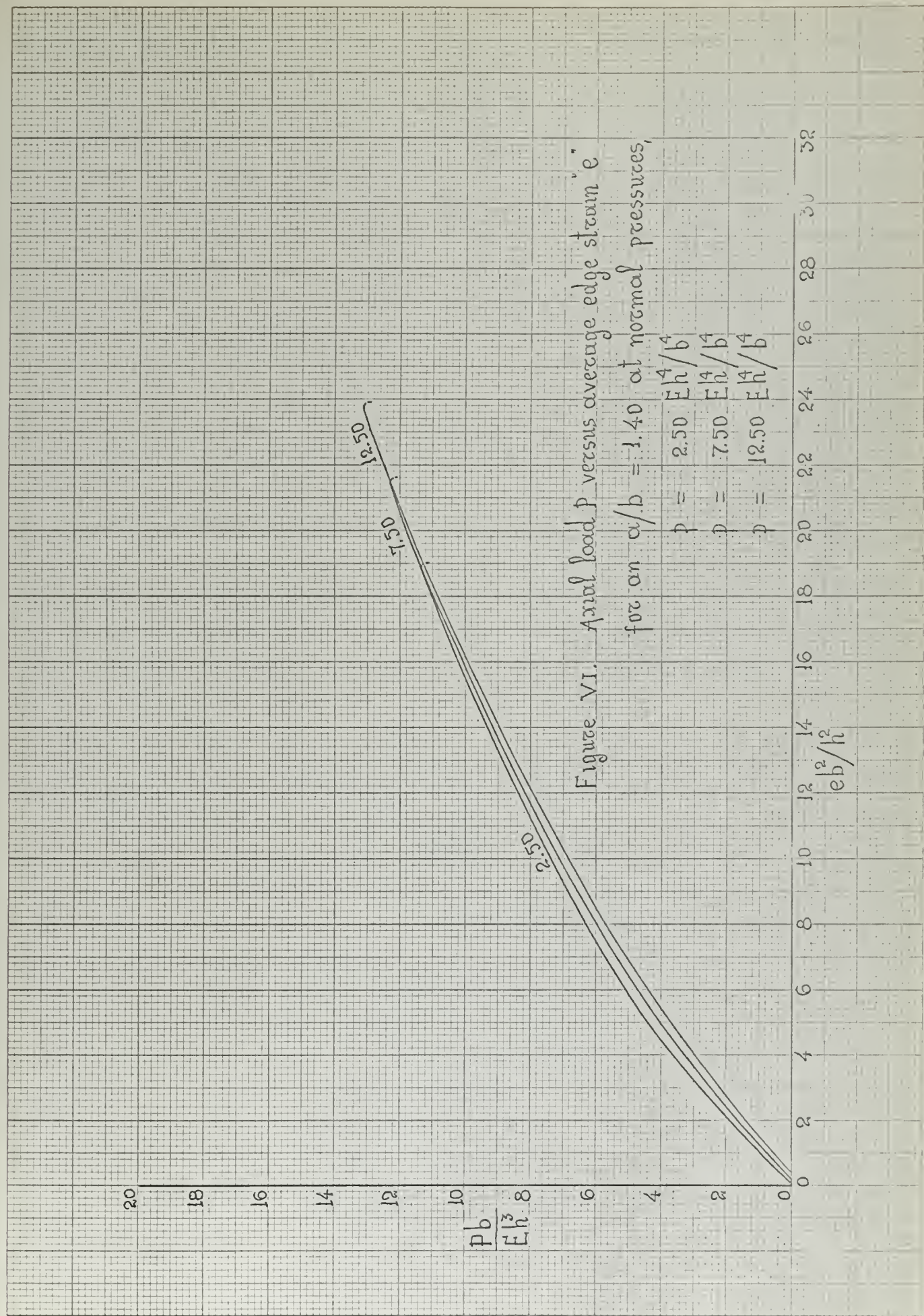


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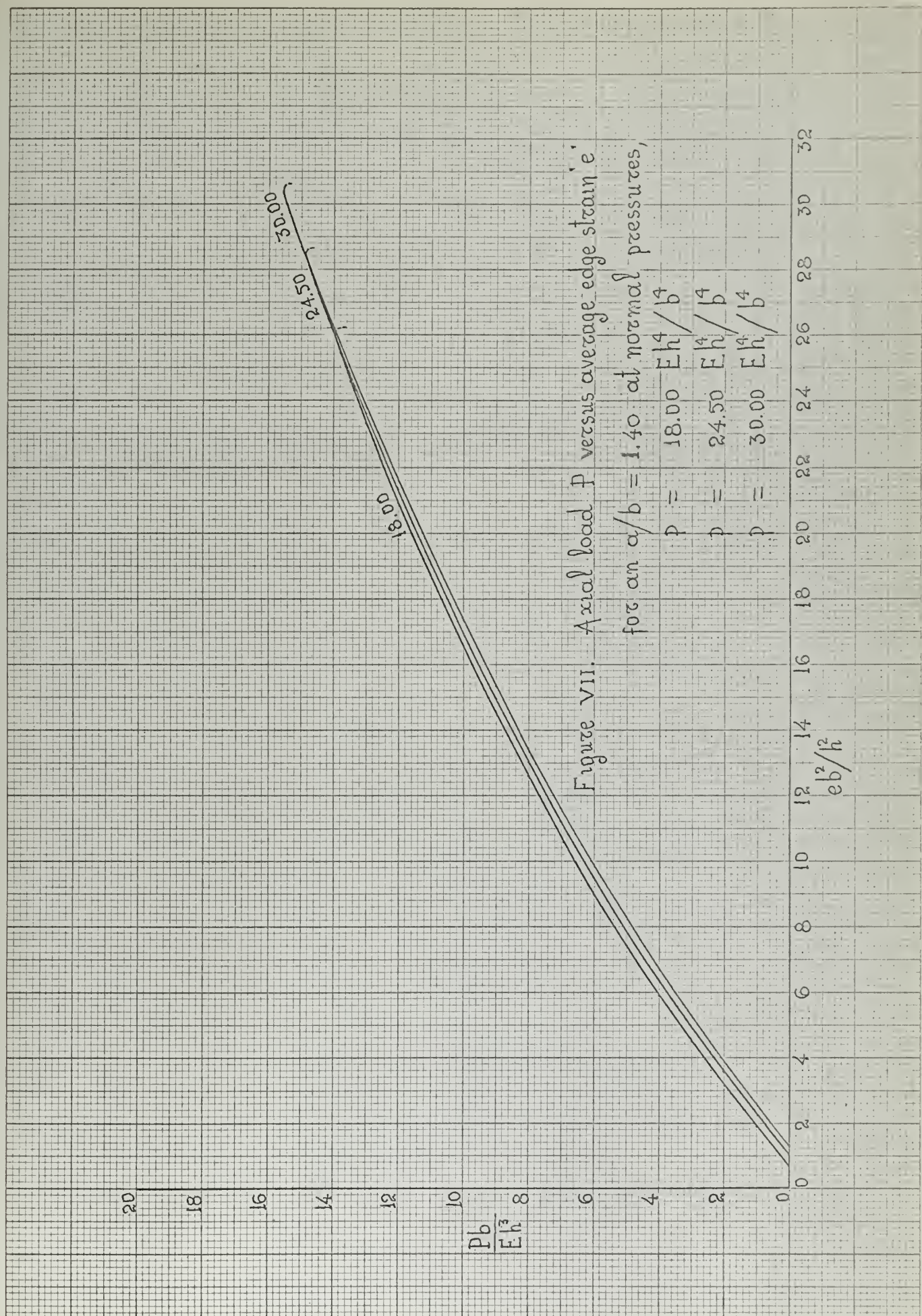


Figure VII. Axial load  $P$  versus average edge strain  $e$

for an  $a/b = 1.40$  at normal pressures,

- $p = 18.00 \frac{Eh^4}{b^4}$
- $p = 24.50 \frac{Eh^4}{b^4}$
- $p = 30.00 \frac{Eh^4}{b^4}$





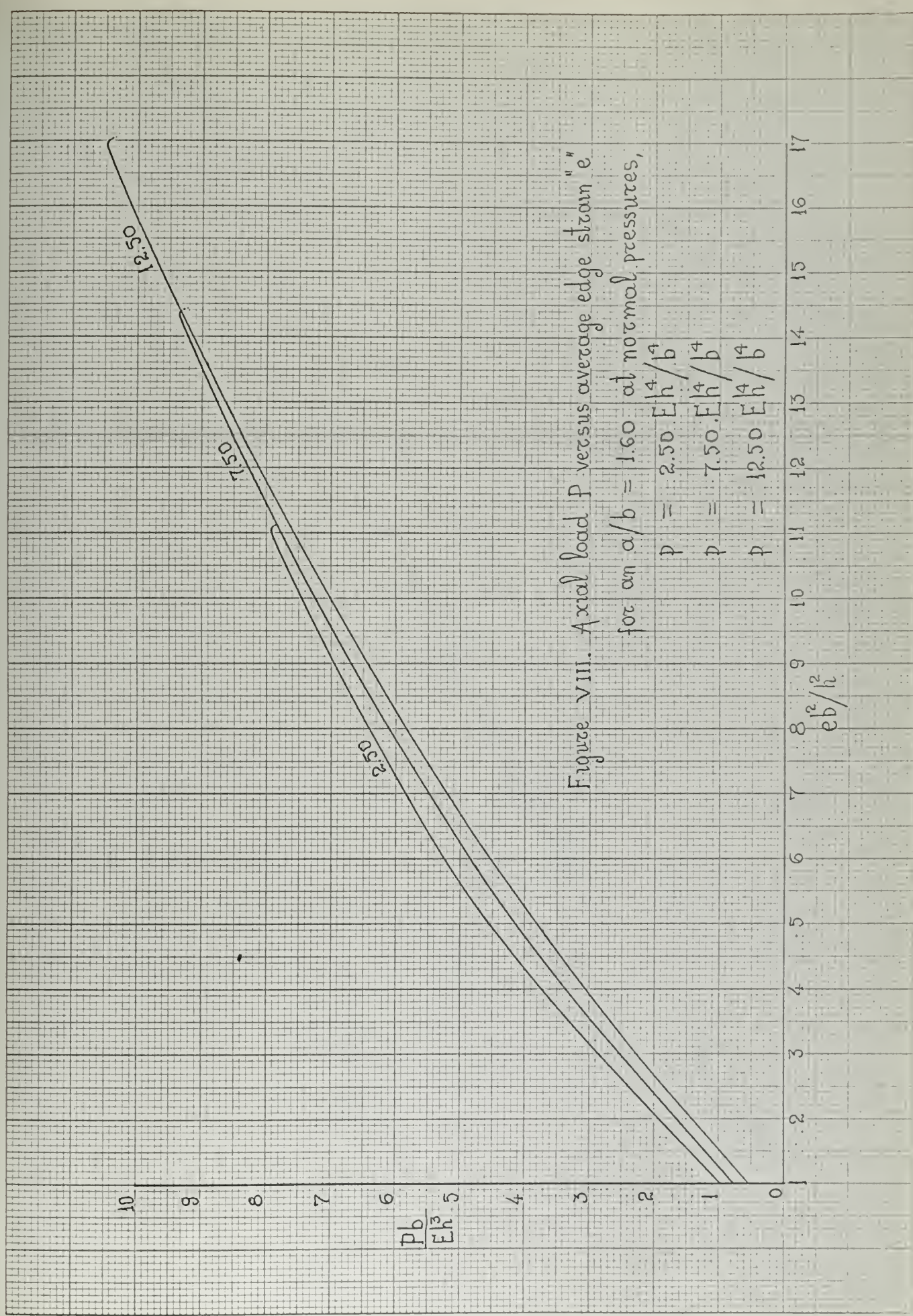


Figure VIII. Axial load  $P$  versus average edge strain  $e$

for an  $a/b = 1.60$  at normal pressures,

$$p = 2.50 \frac{Eh^4}{b^4}$$

$$p = 7.50 \frac{Eh^4}{b^4}$$

$$p = 12.50 \frac{Eh^4}{b^4}$$





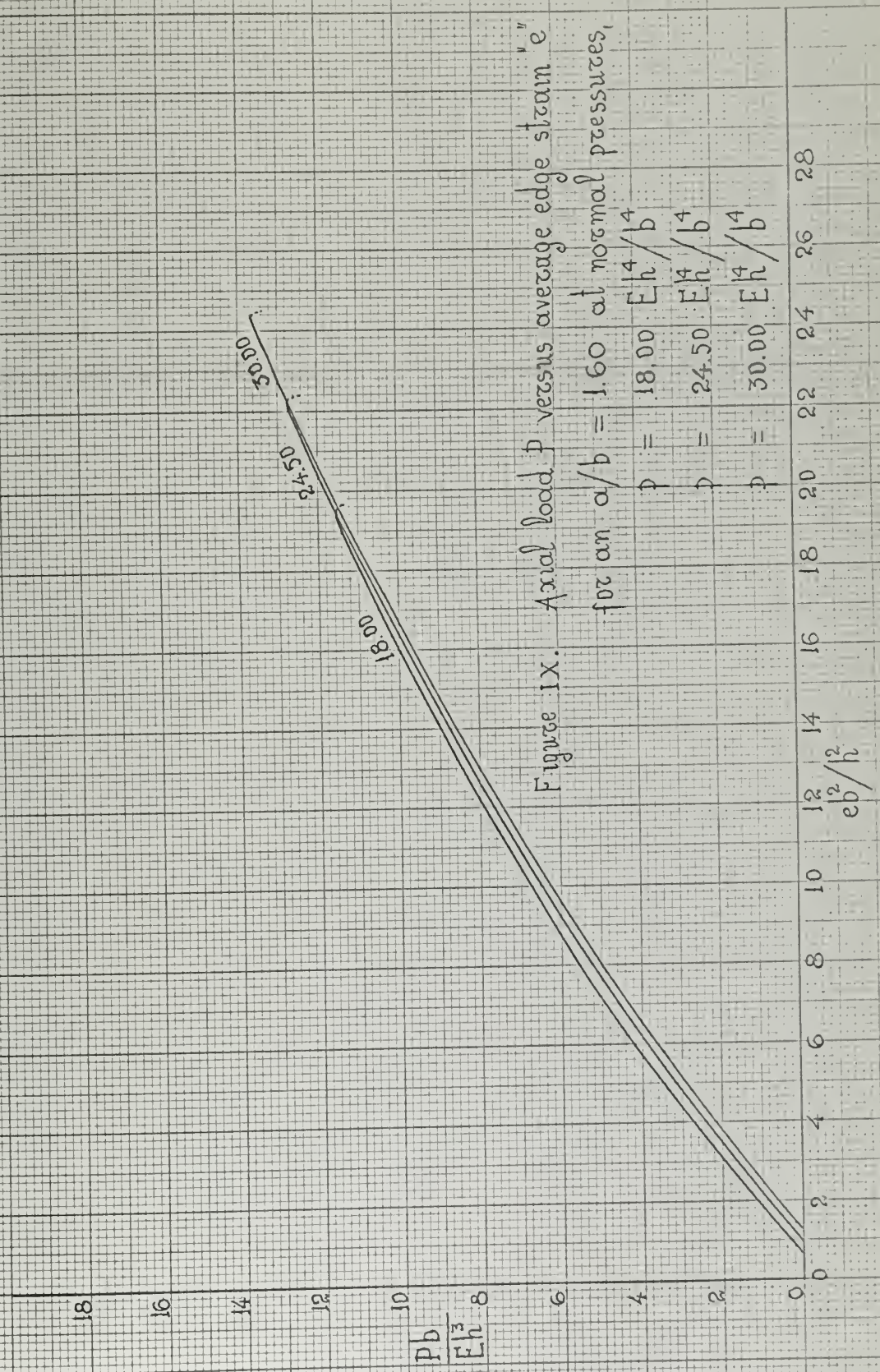
Figure 1X. Axial load  $P$  versus average edge strain  $e$

for an  $a/b = 1.60$  at normal pressures,

$P = 18.00 \frac{Eh^4}{b^4}$

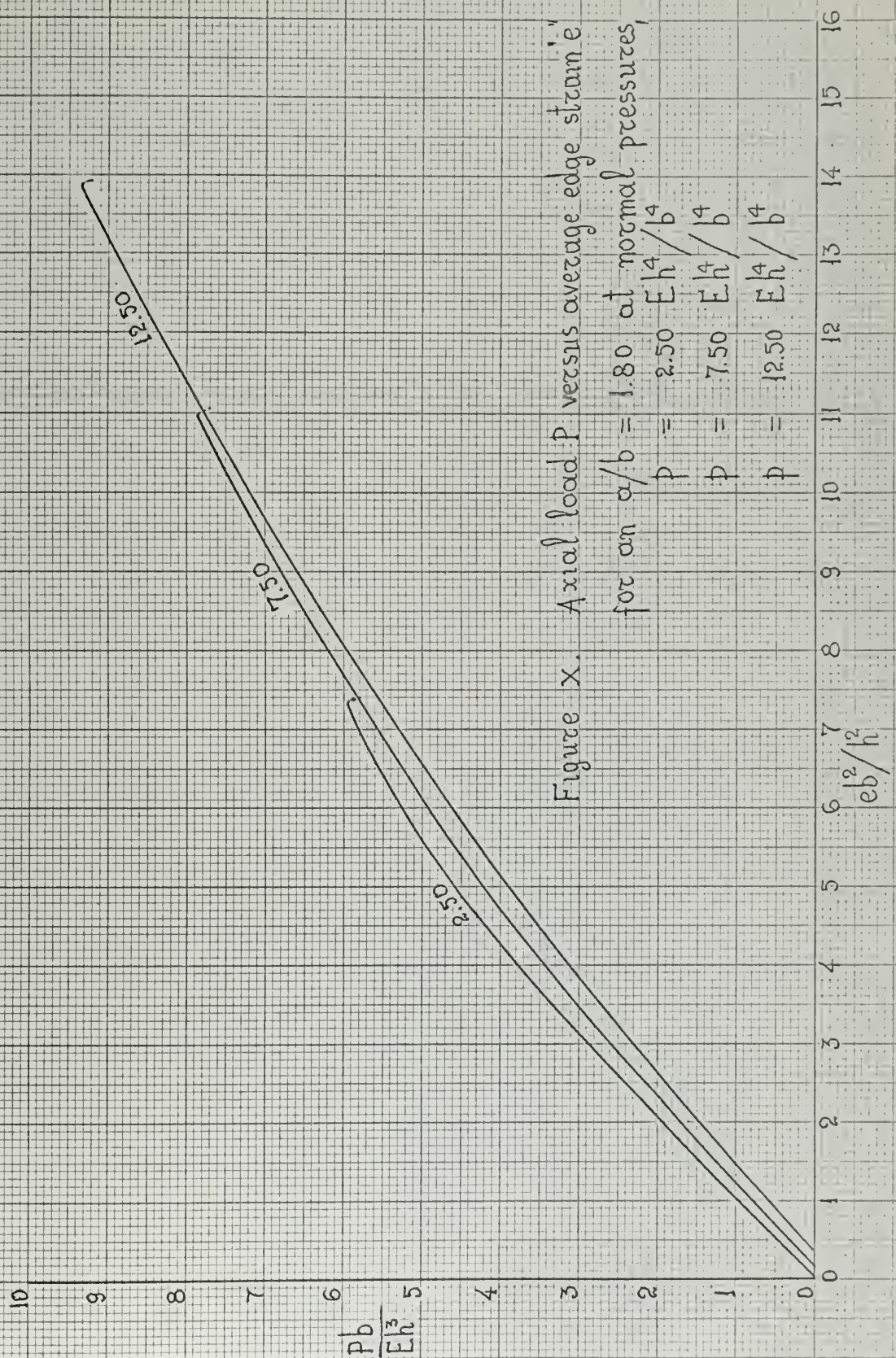
$P = 24.50 \frac{Eh^4}{b^4}$

$P = 30.00 \frac{Eh^4}{b^4}$



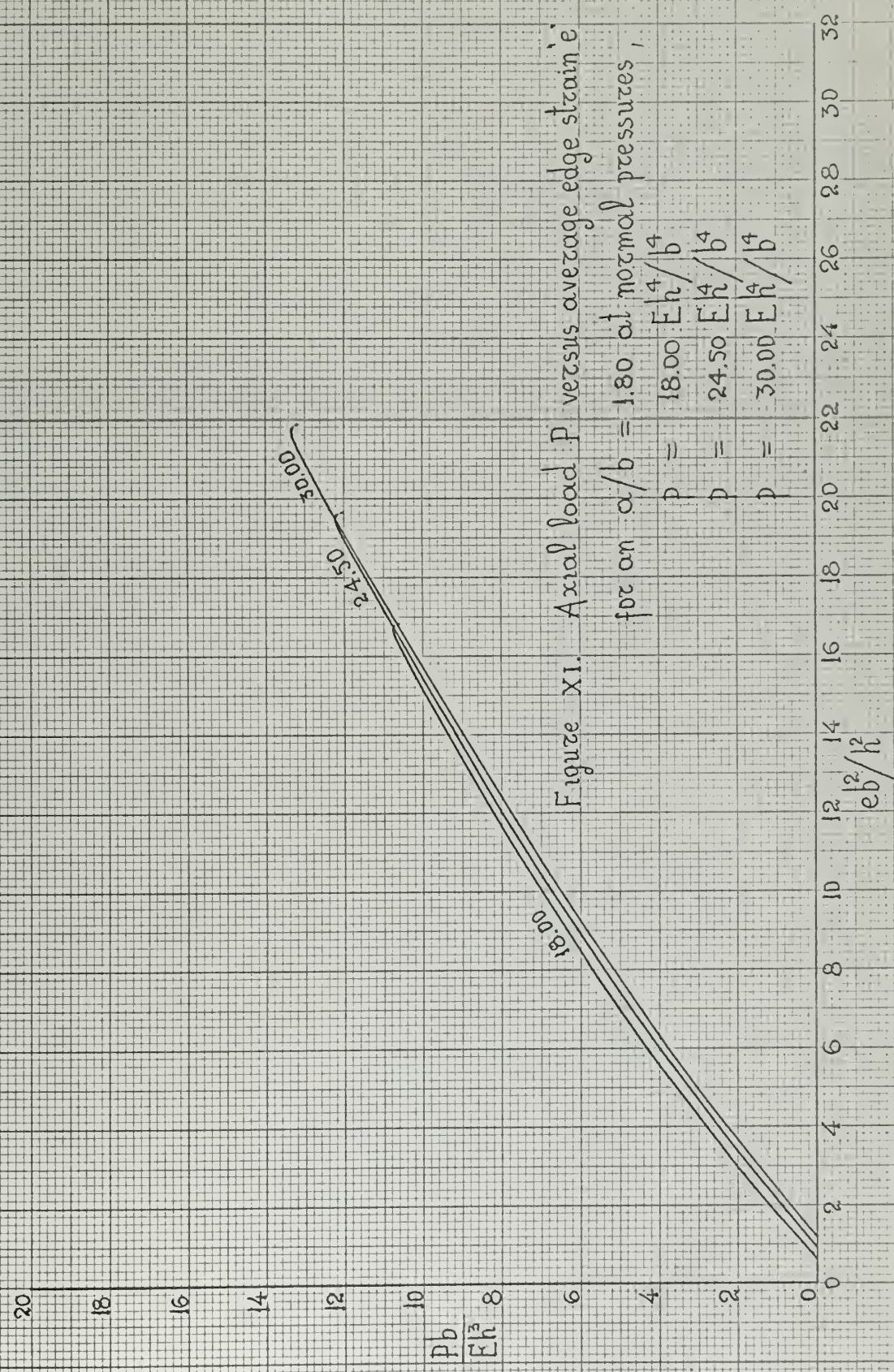






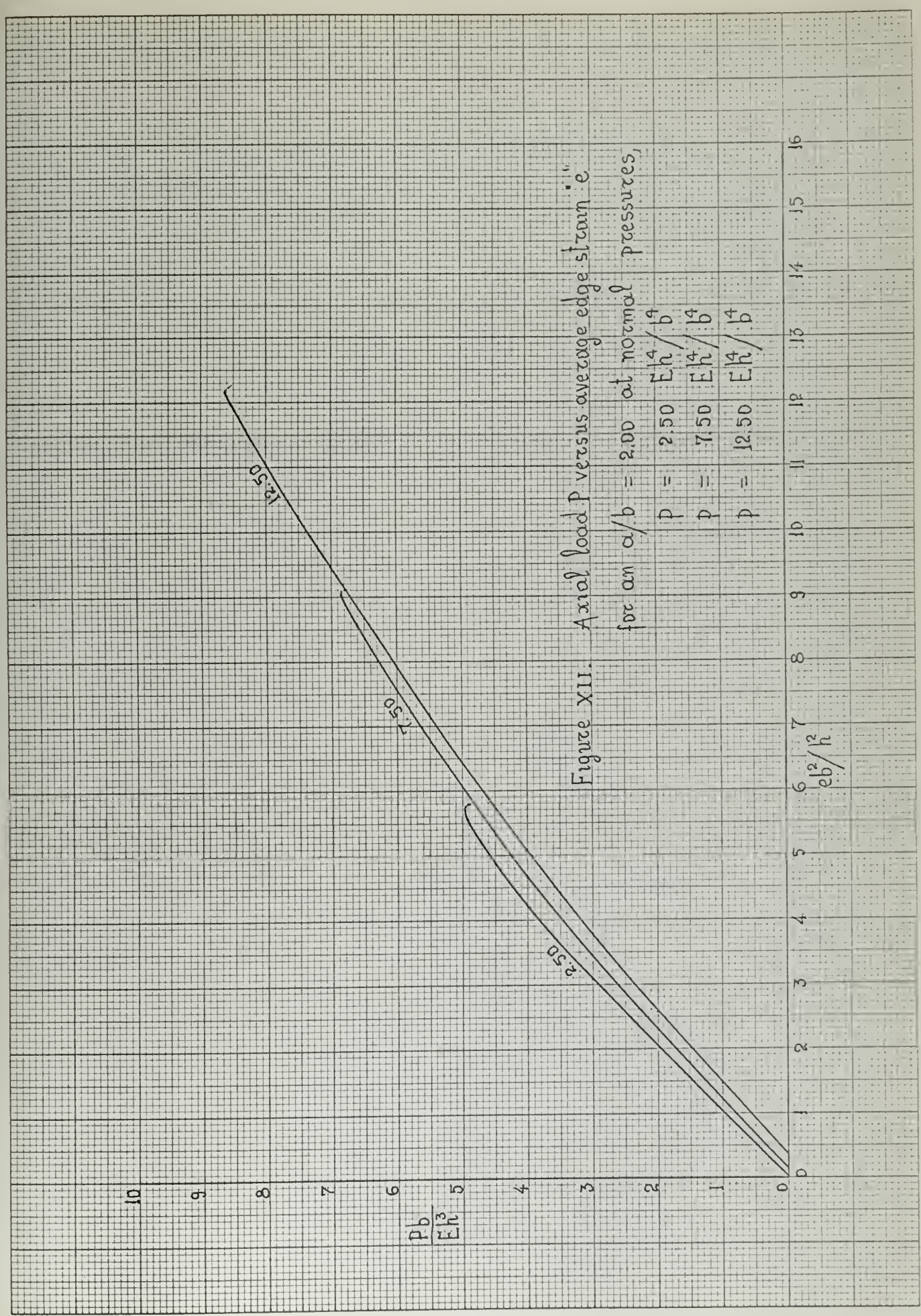






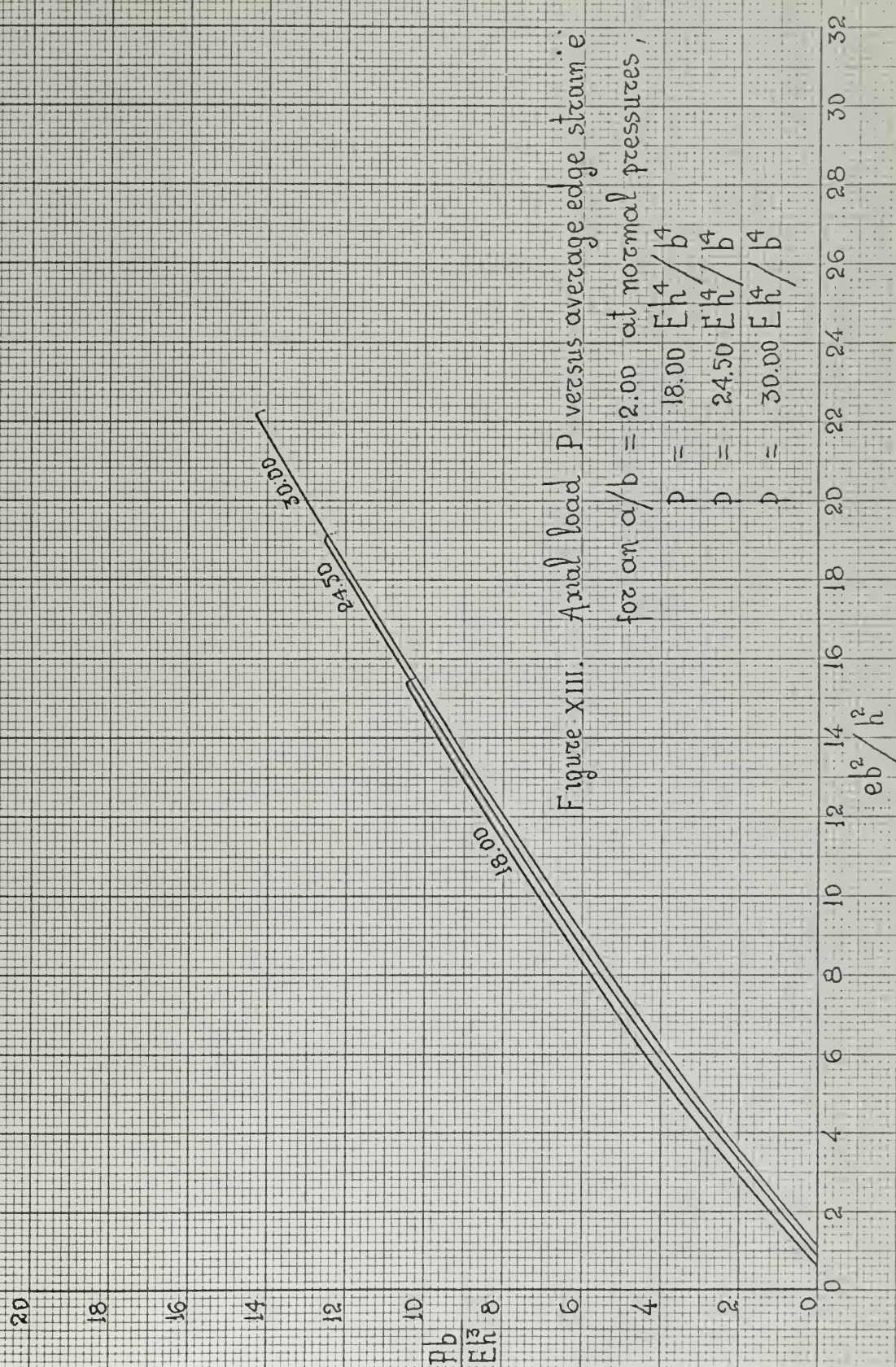
















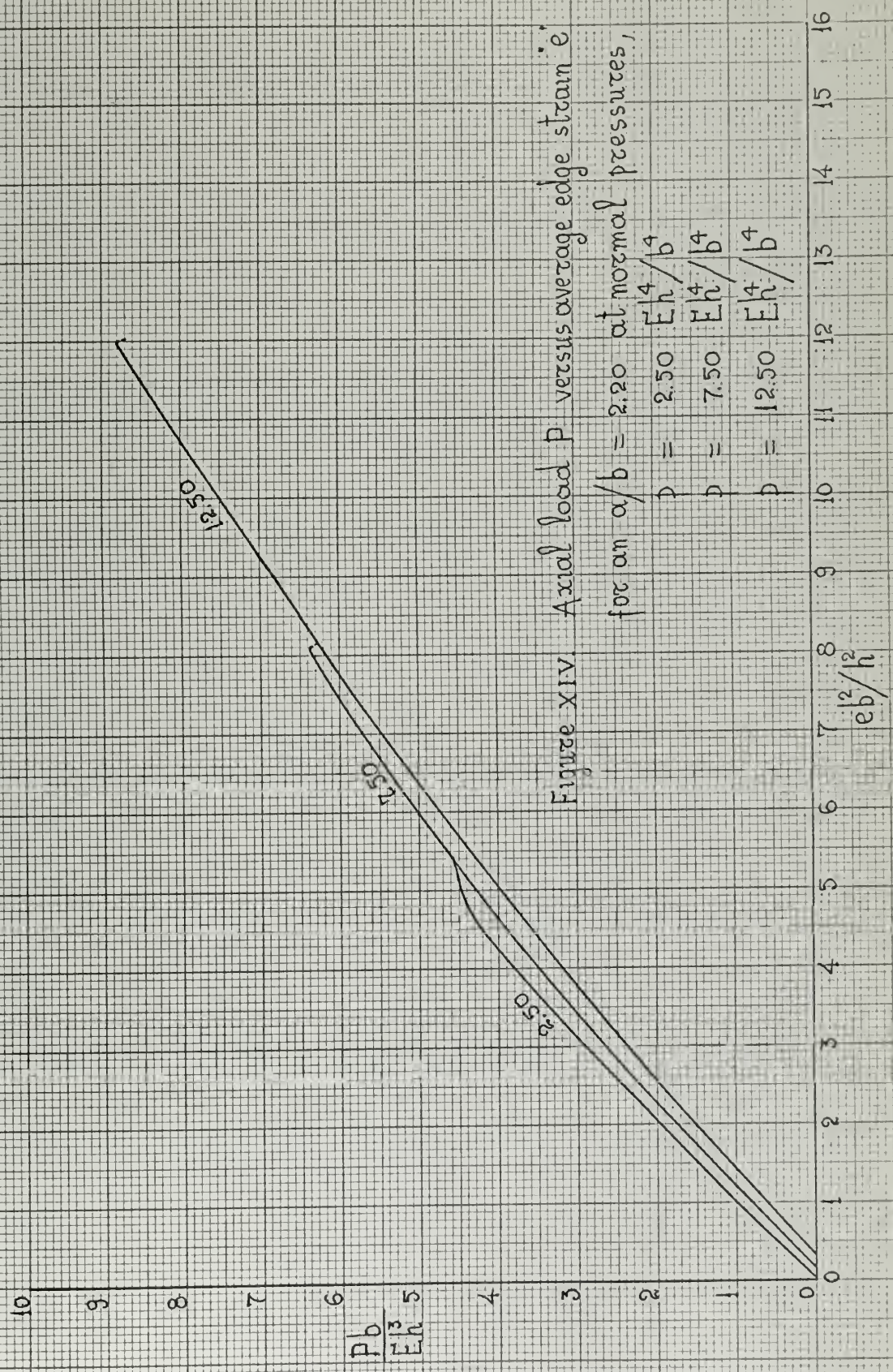
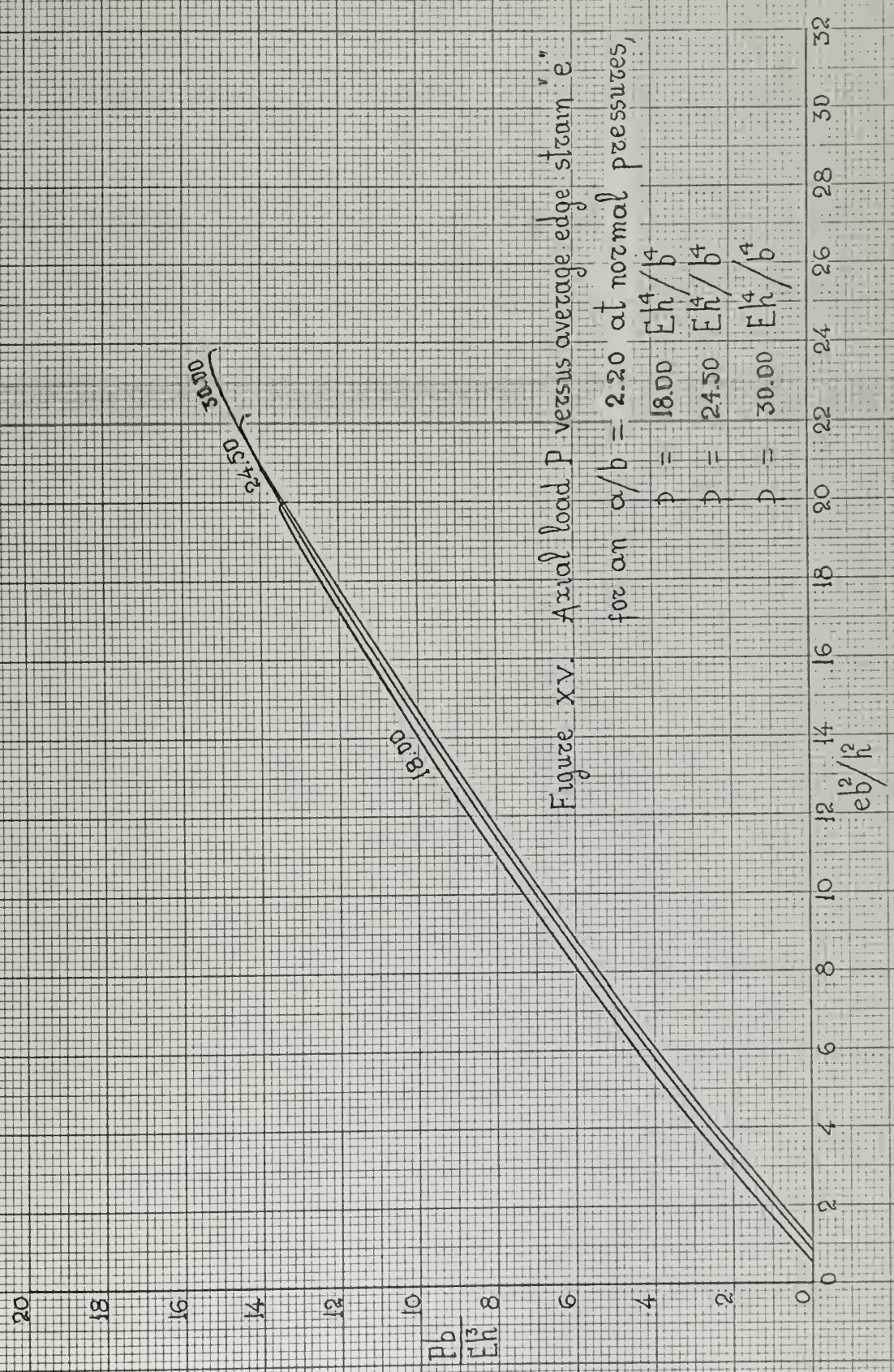


Figure XIV. Axial load  $P$  versus average edge strain  $e$

for an  $a/b = 2.20$  at normal pressures,  
 $p = \frac{Eh^4}{b^4}$   
 $p = \frac{Eh^4}{b^4}$   
 $p = \frac{Eh^4}{b^4}$











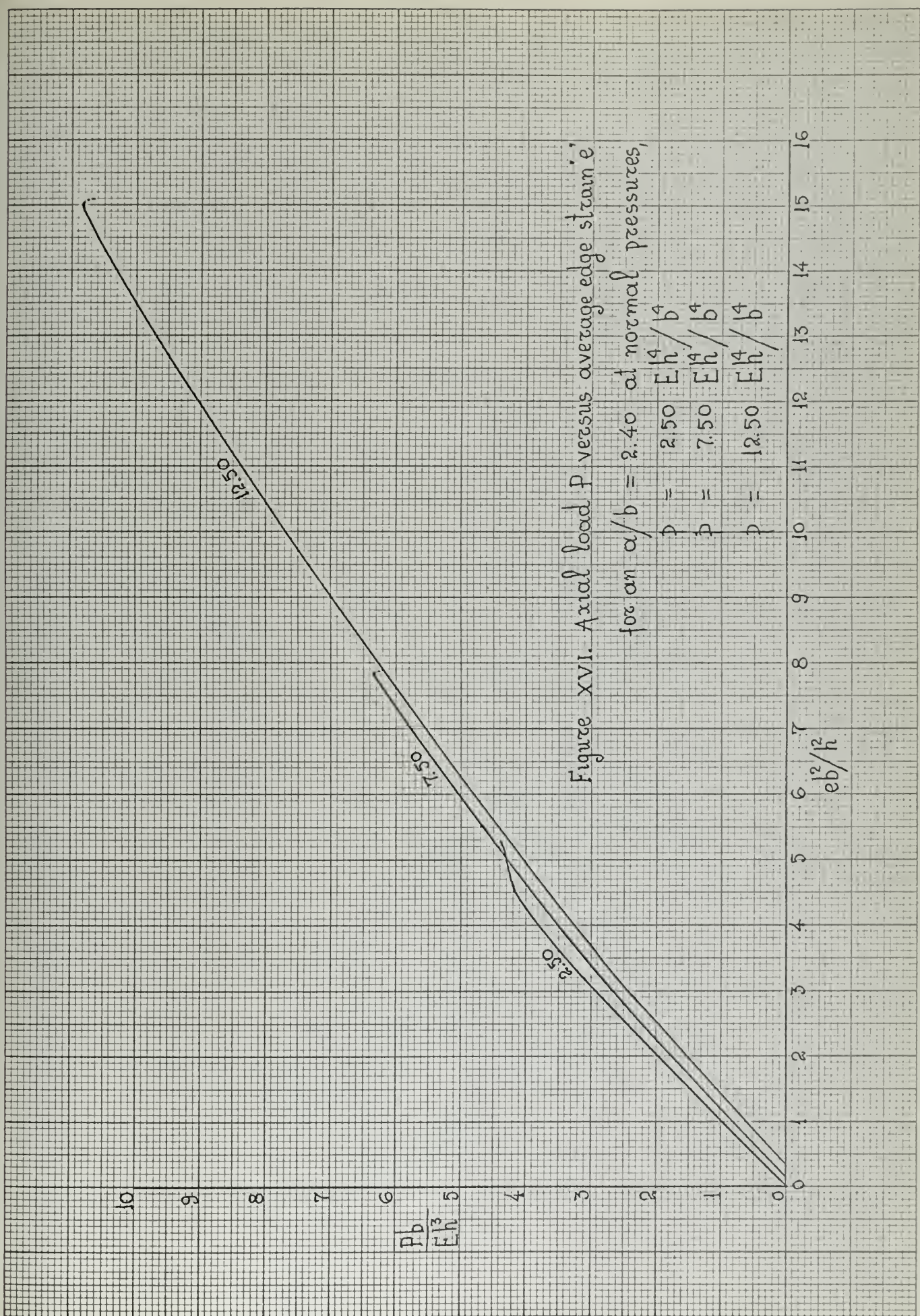


Figure XVI. Axial load  $P$  versus average edge strain  $e$ , for an  $a/b = 2.40$  at normal pressures,

$\frac{Eh^4}{b^4} = 2.50$   
 $\frac{Eh^4}{b^4} = 7.50$   
 $\frac{Eh^4}{b^4} = 12.50$





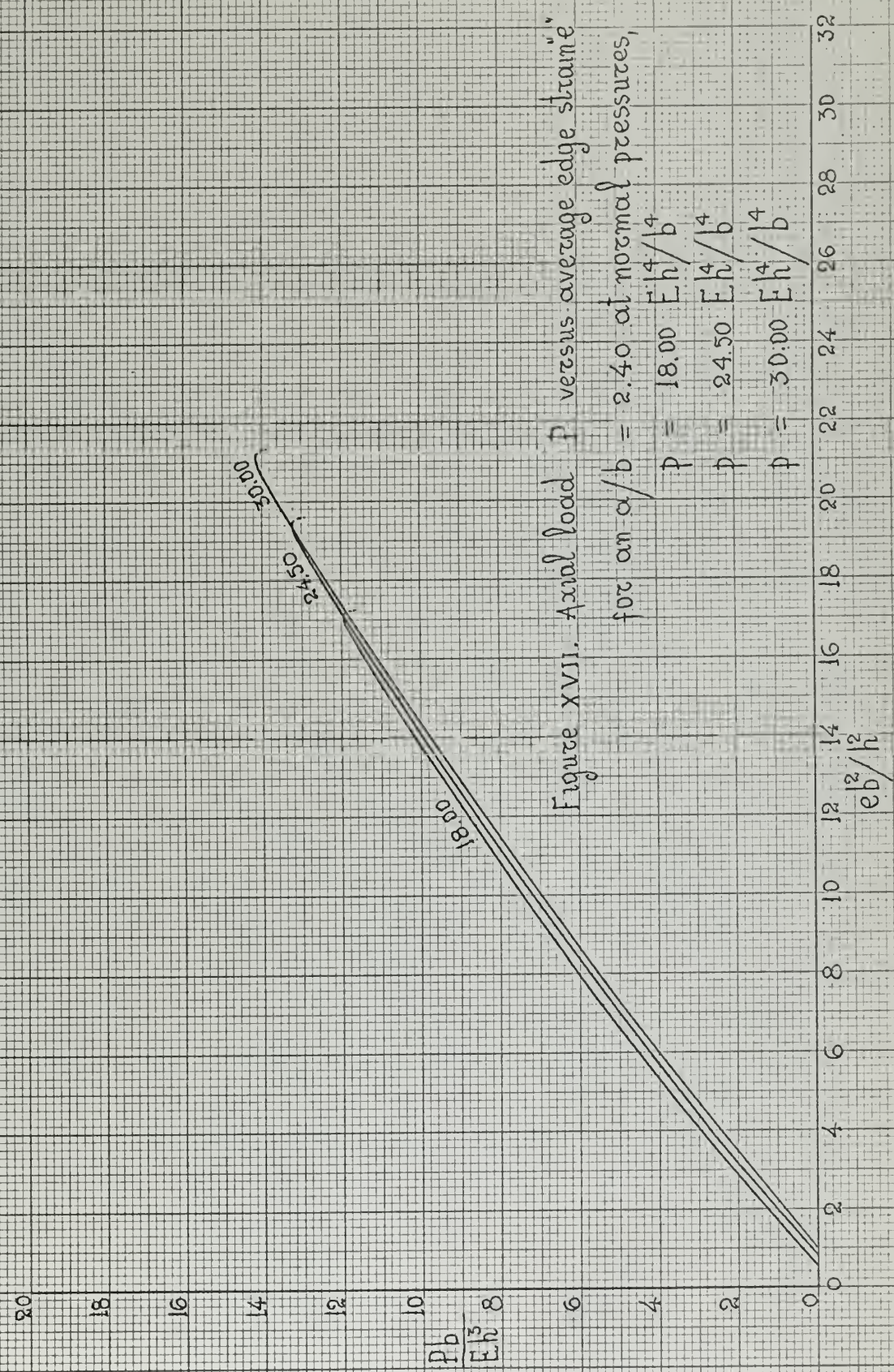
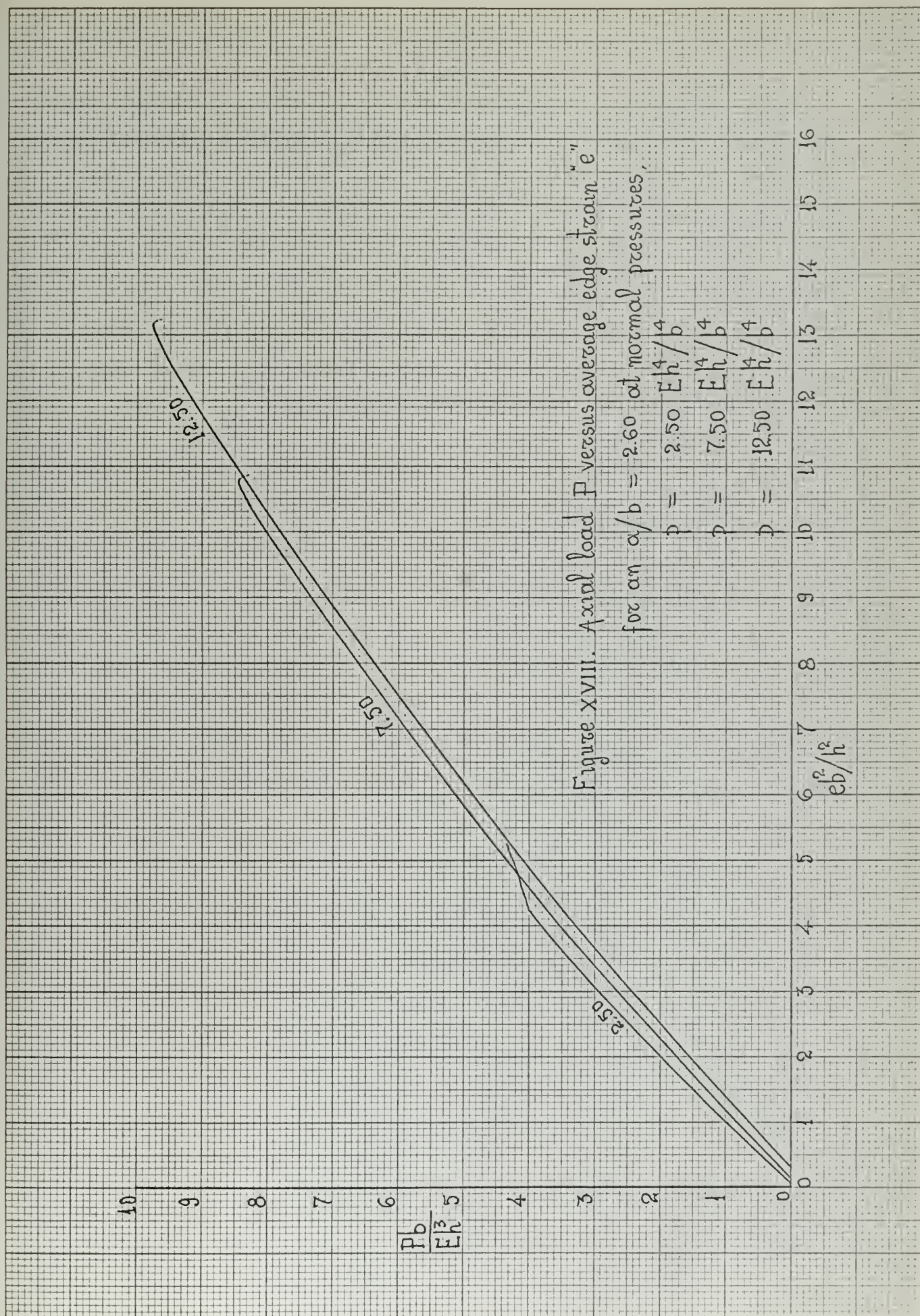


Figure XVII. Axial load  $P$  versus average edge strain

for an  $\alpha/b = 2.40$  at normal pressures,  
 $p = 18.00 \frac{Eh^4}{b^4}$   
 $p = 24.50 \frac{Eh^4}{b^4}$   
 $p = 30.00 \frac{Eh^4}{b^4}$

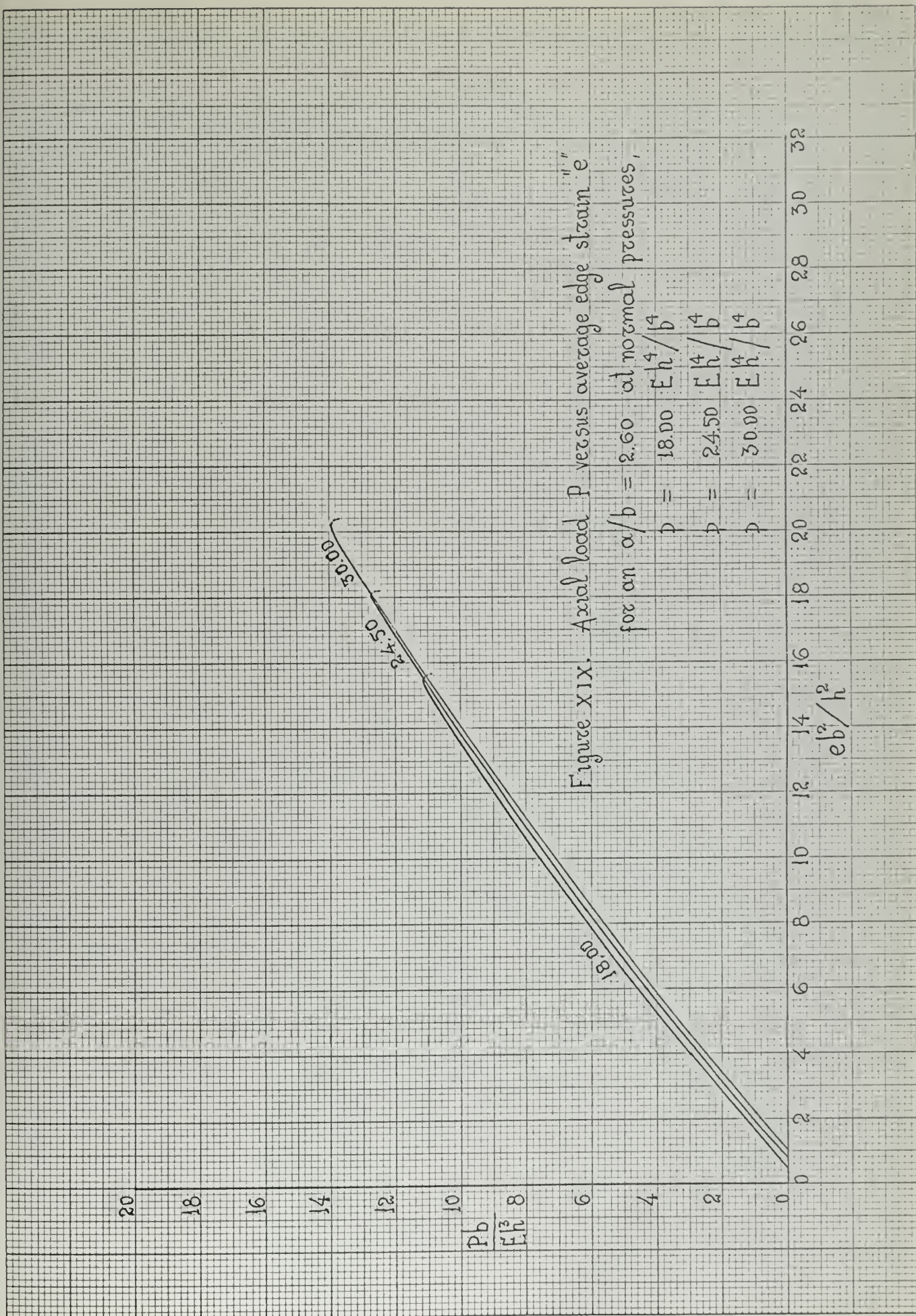
















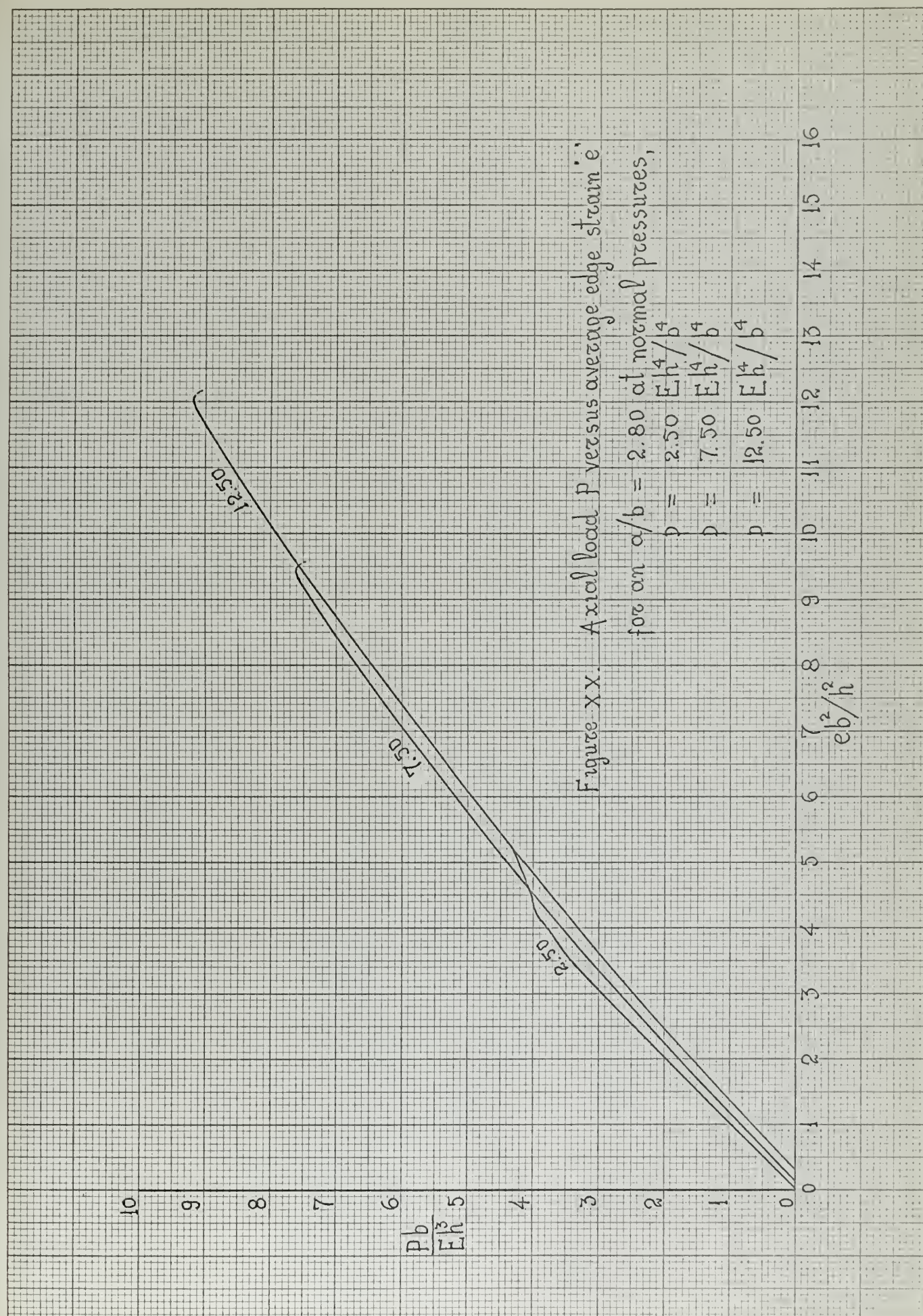


Figure XX. Axial load  $P$  versus average edge strain  $e$

for an  $a/b = 2.80$  at normal pressures,

$$p = \frac{Ph^4}{Eb^4}$$

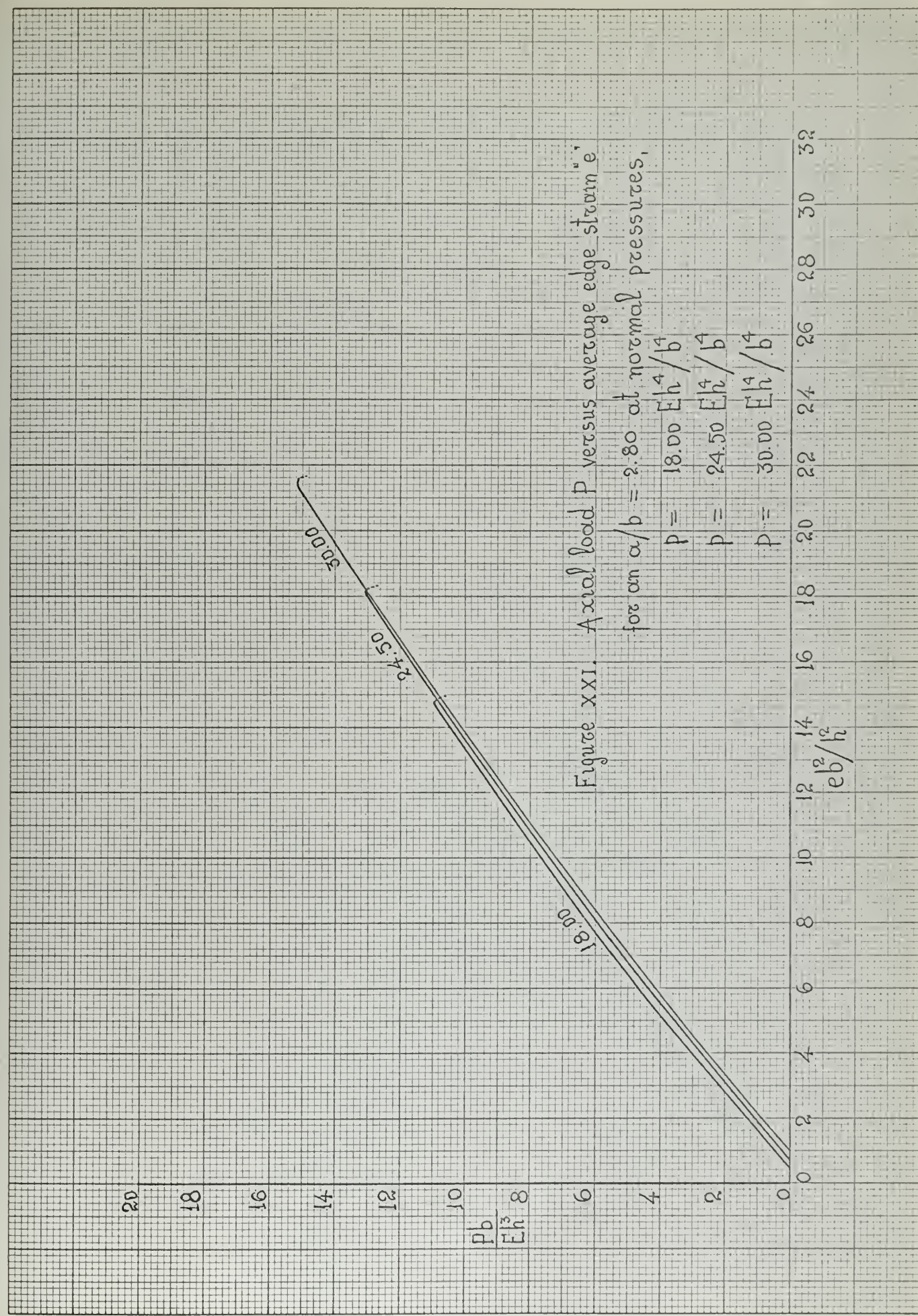
$$p = \frac{Ph^4}{Eb^4}$$

$$p = \frac{Ph^4}{Eb^4}$$

$$p = \frac{Ph^4}{Eb^4}$$











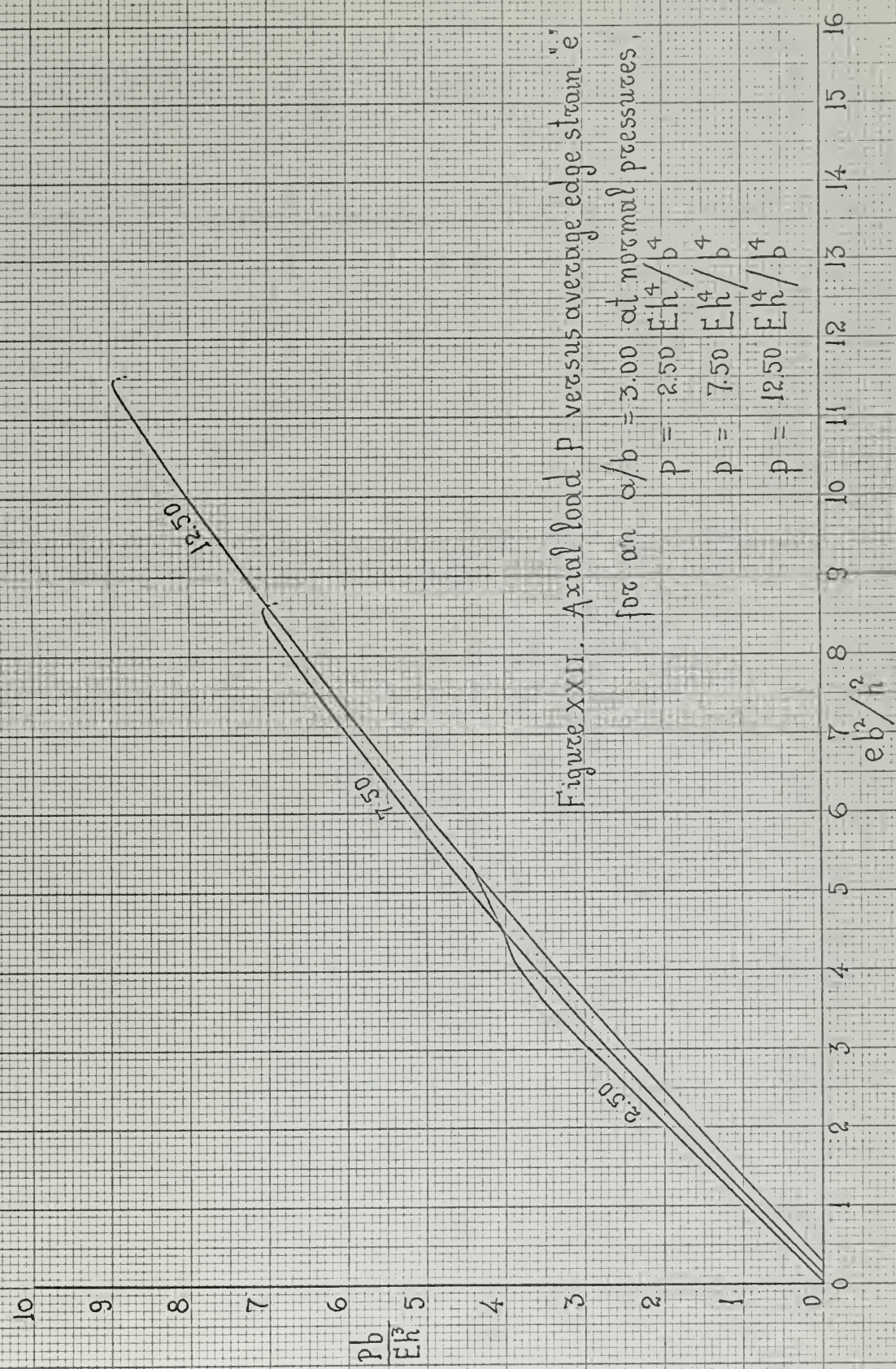


Figure XXII. Axial load  $P$  versus average edge strain  $e$  for an  $\alpha/b = 3.00$  at normal pressures,  
 $P = 2.50 \frac{E h^4}{b^4}$   
 $P = 7.50 \frac{E h^4}{b^4}$   
 $P = 12.50 \frac{E h^4}{b^4}$





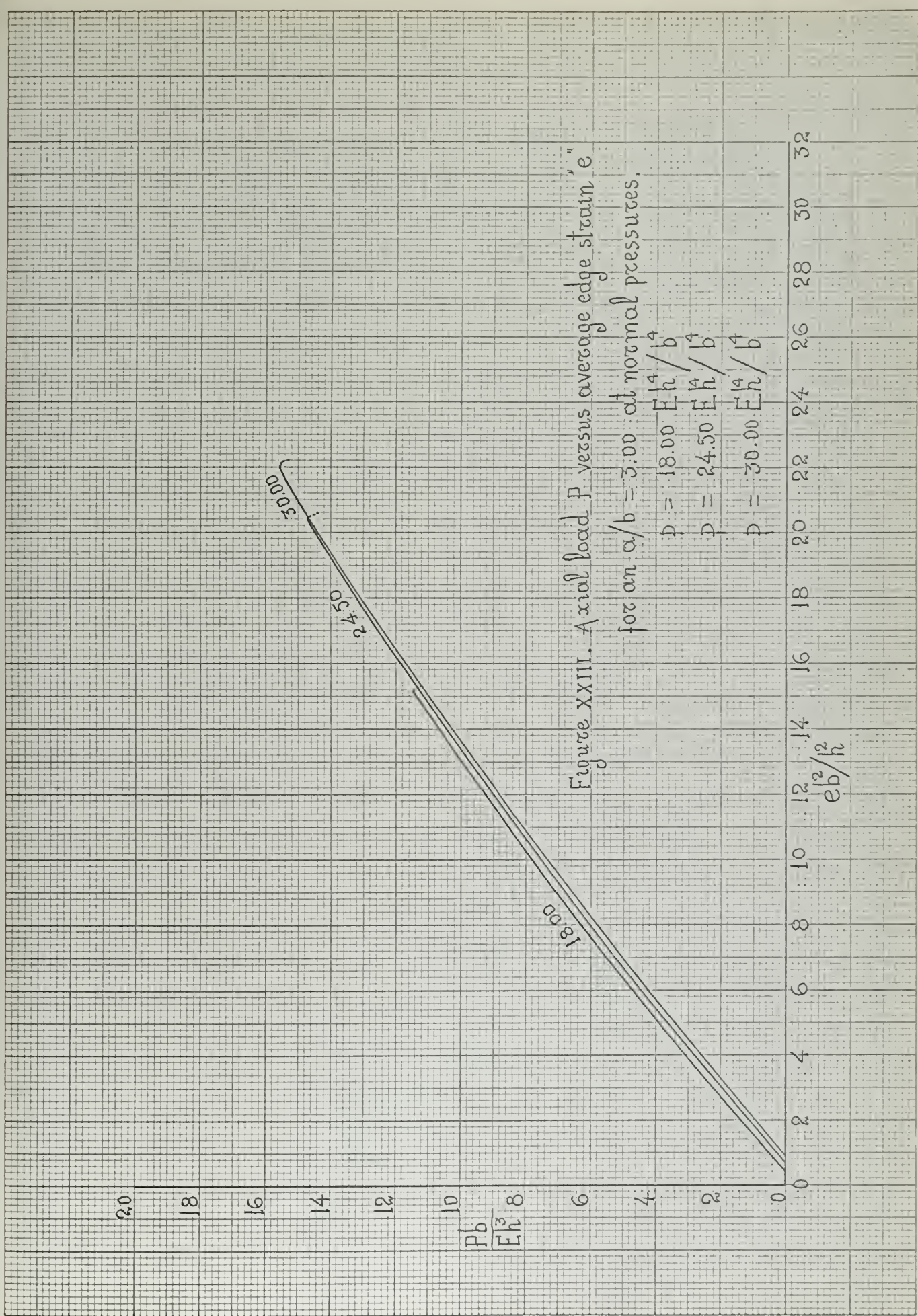
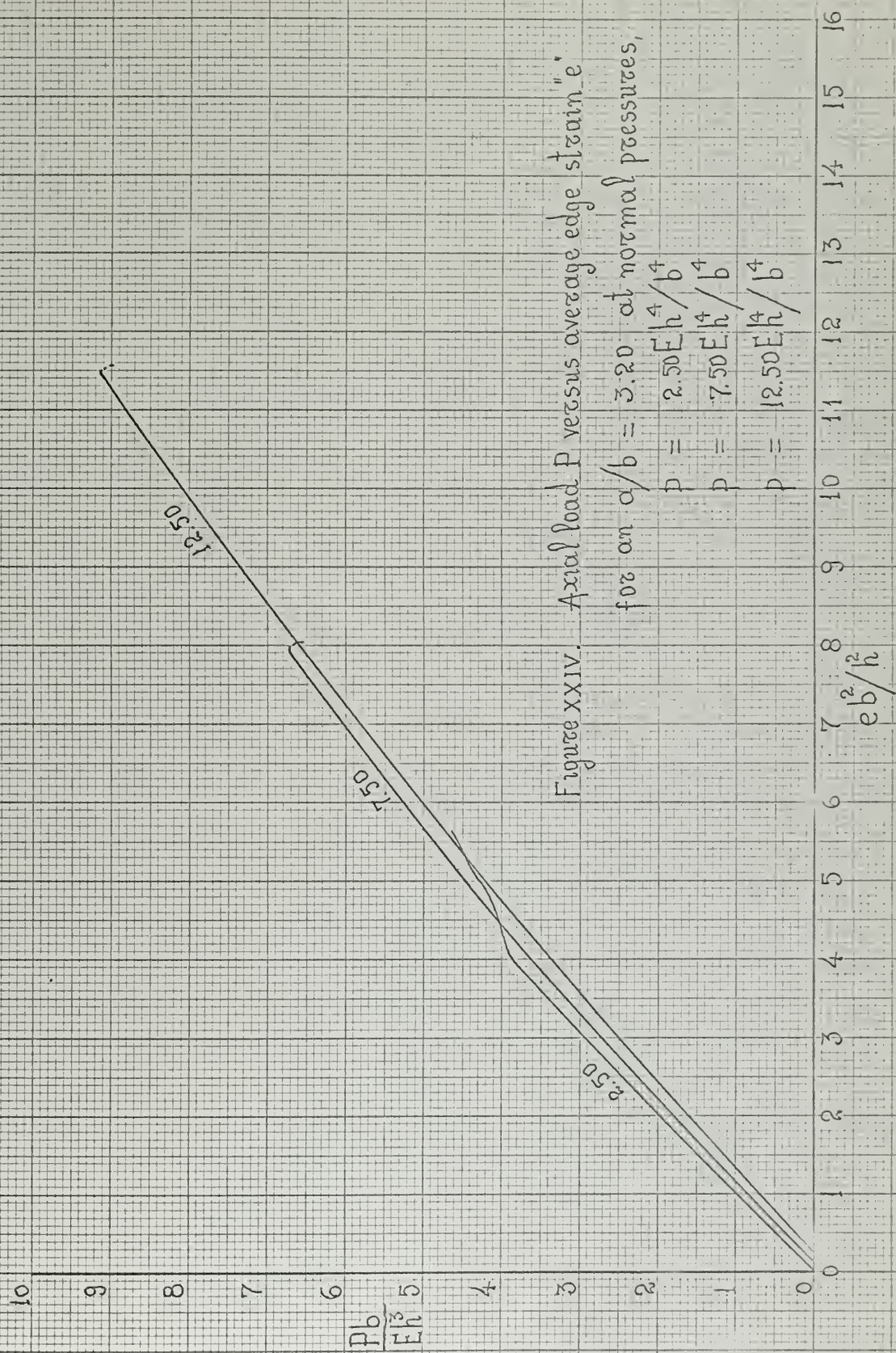


Figure XXIII. Axial load  $P$  versus average edge strain "e" for an  $\alpha/b = 3.00$  at normal pressures.

$\frac{P}{Eh^3/b^4} = 18.00$   
 $\frac{P}{Eh^3/b^4} = 24.50$   
 $\frac{P}{Eh^3/b^4} = 30.00$











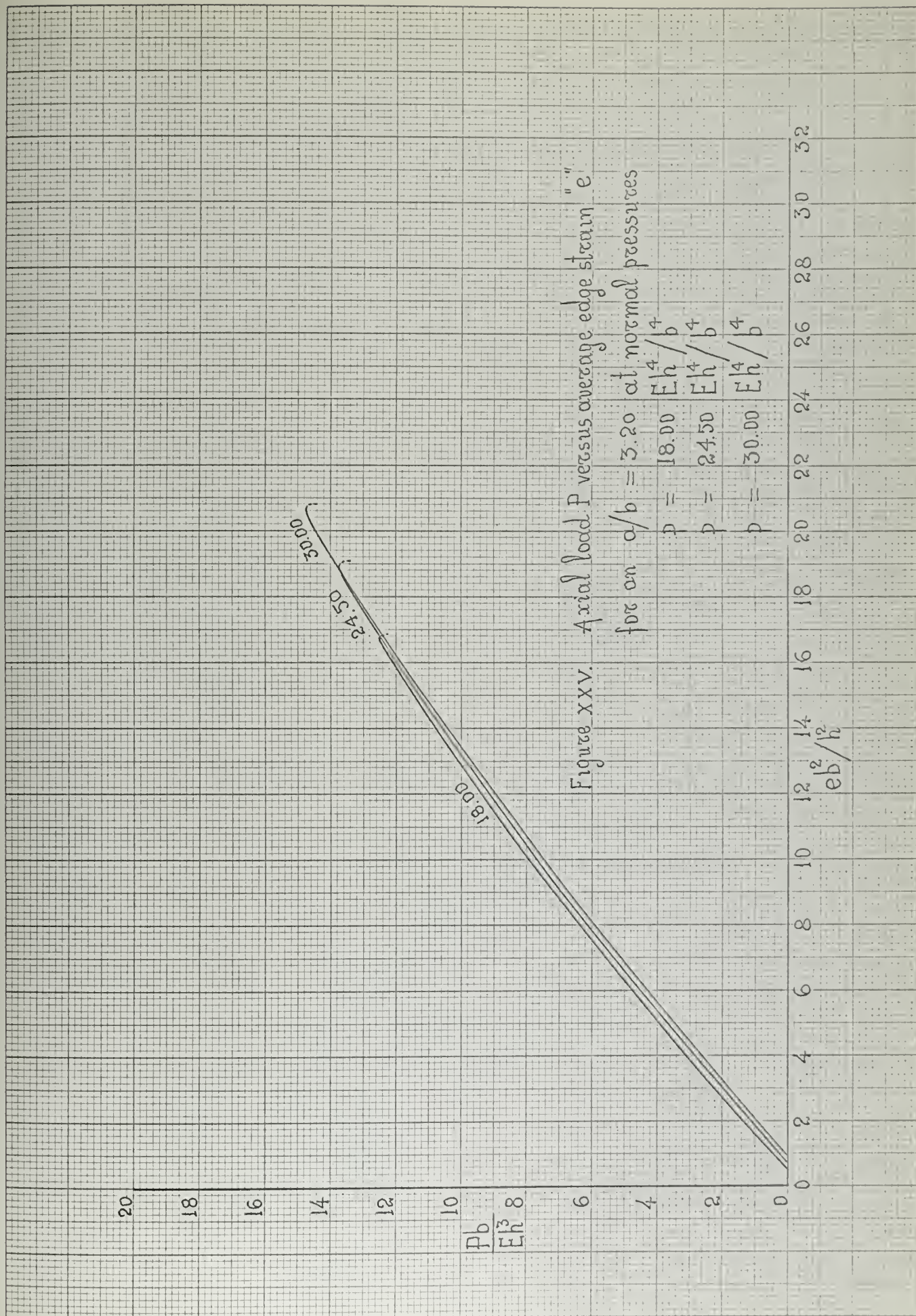


Figure XXV. Axial load  $P$  versus average edge strain  $e$  for on  $a/b = 3.20$  at normal pressures

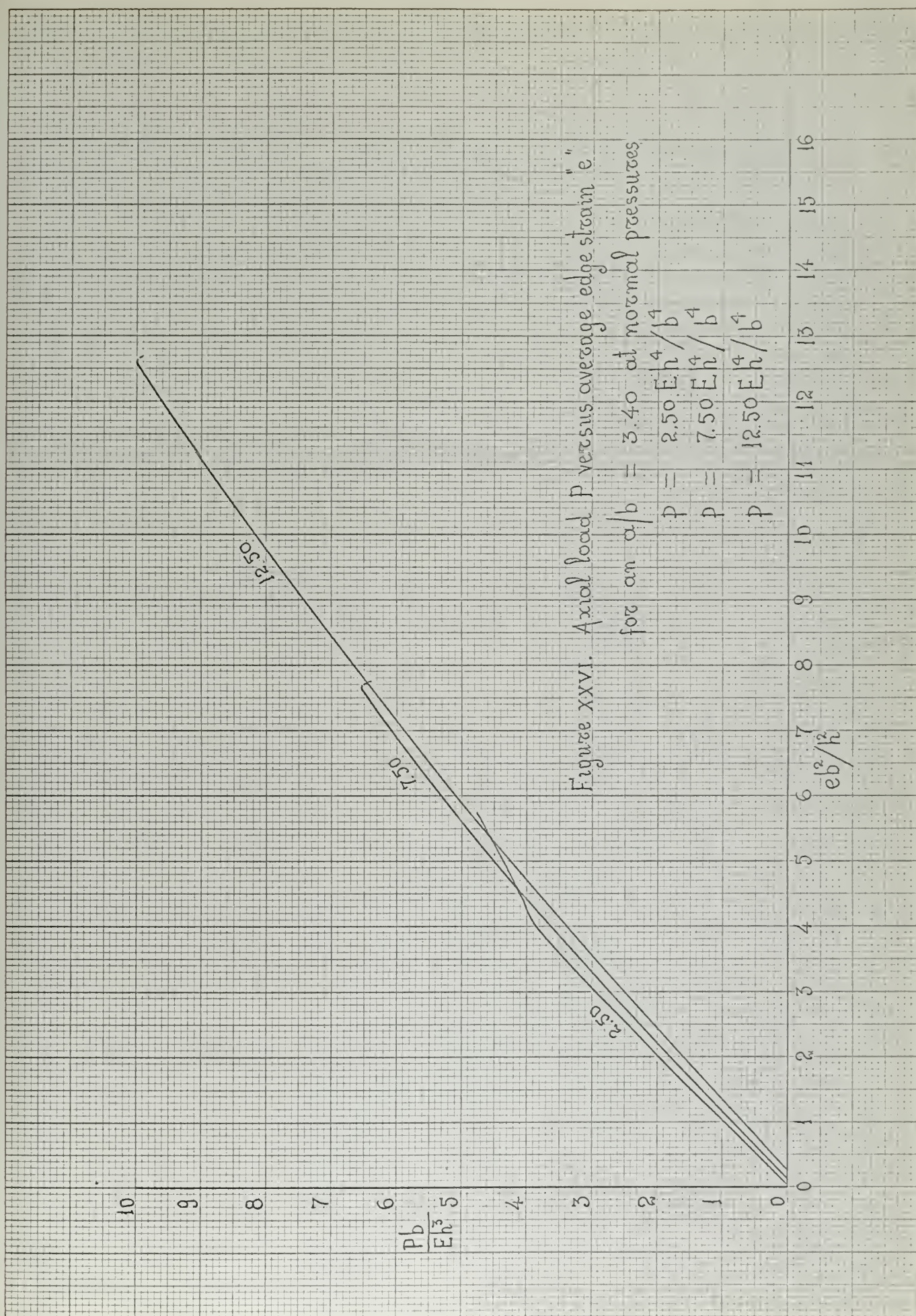
$p = 18.00$   
 $p = 24.50$   
 $p = 30.00$

$\frac{Pb}{EI^3}$

$\frac{eb^2}{h^2}$











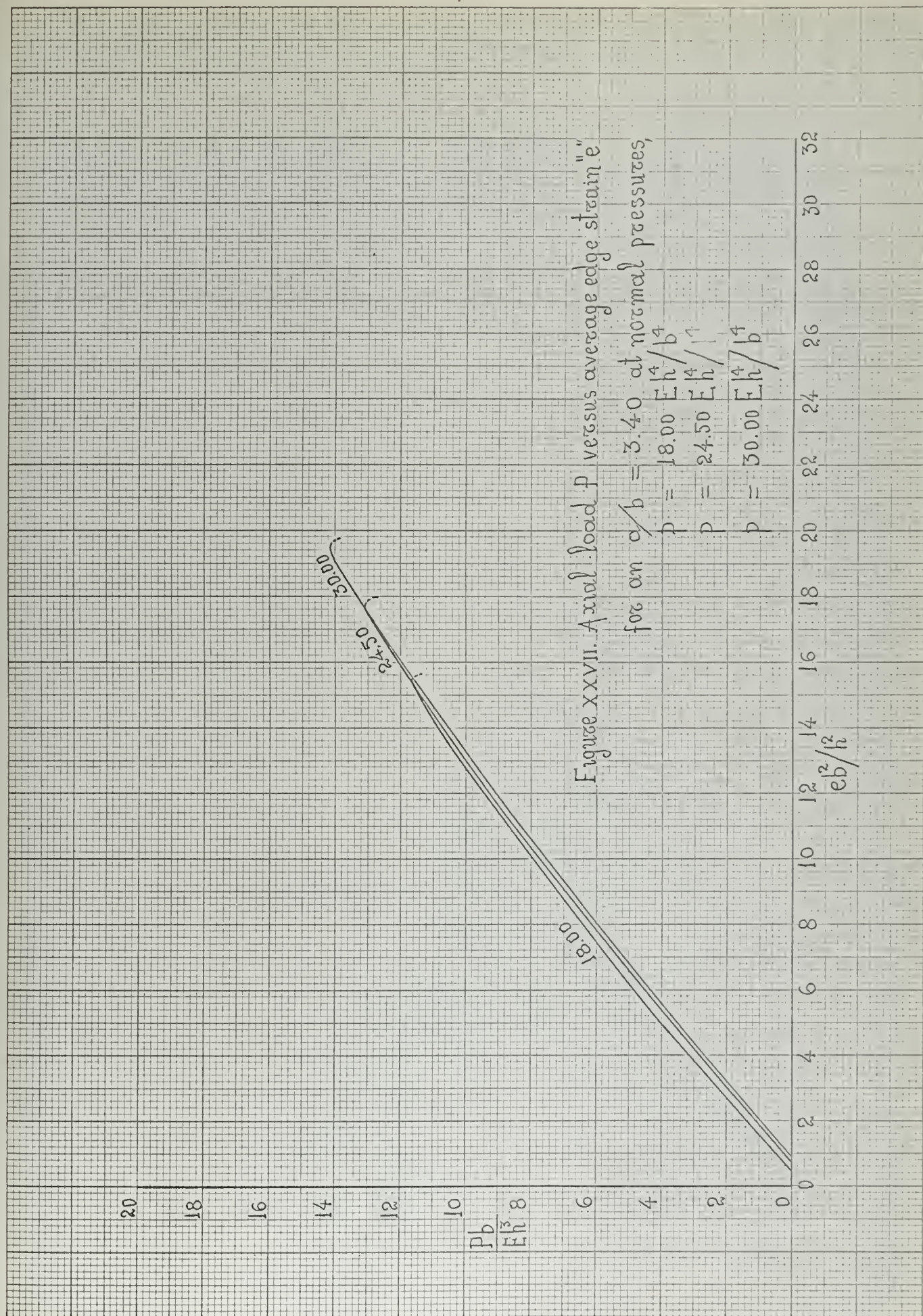


Figure xxvii. Axial load  $P$  versus average edge strain  $e$

for an  $a/b = 3.40$  at normal pressures,

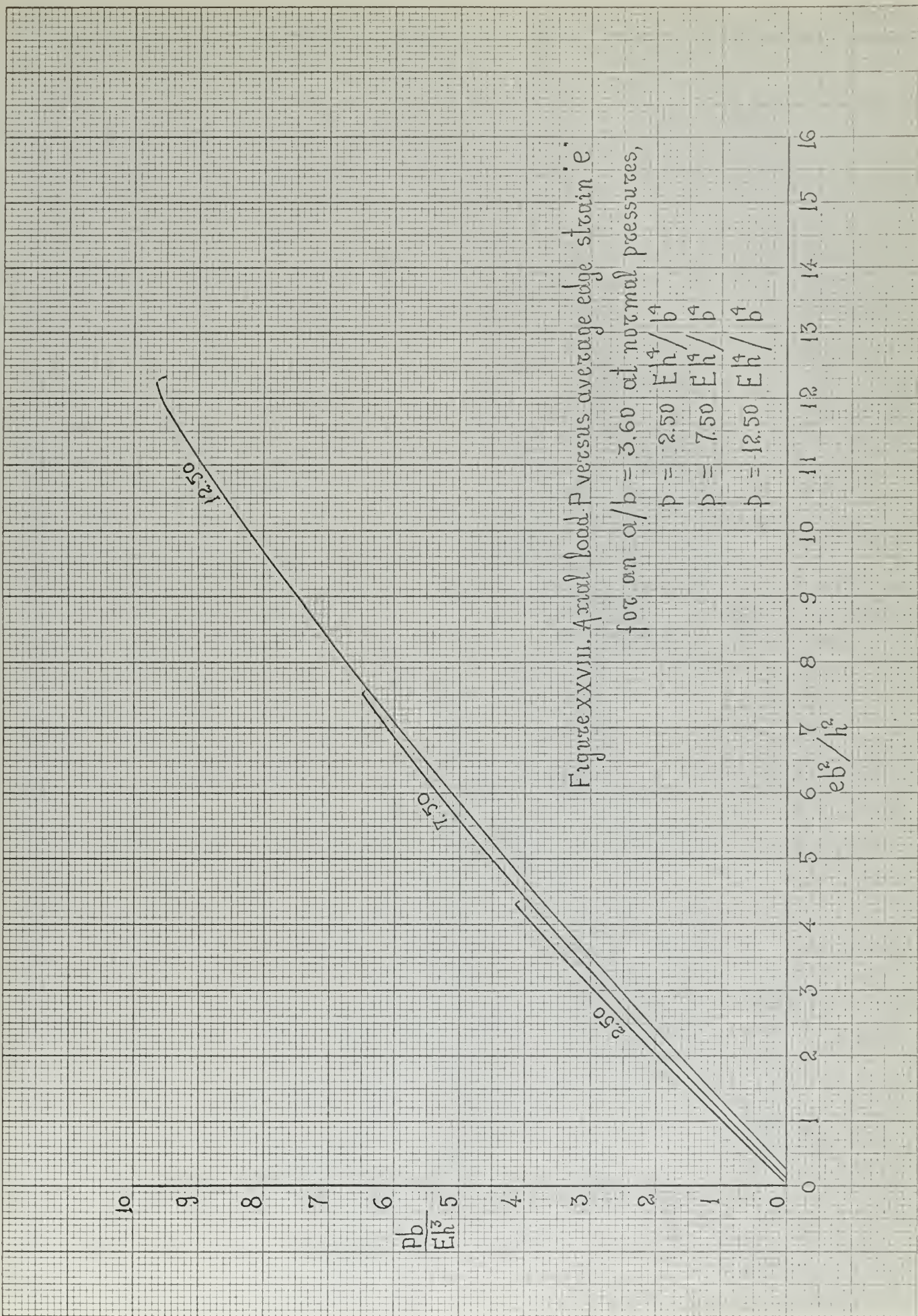
$$p = 18.00 \frac{Eh^4}{b^4}$$

$$p = 24.50 \frac{Eh^4}{b^4}$$

$$p = 30.00 \frac{Eh^4}{b^4}$$

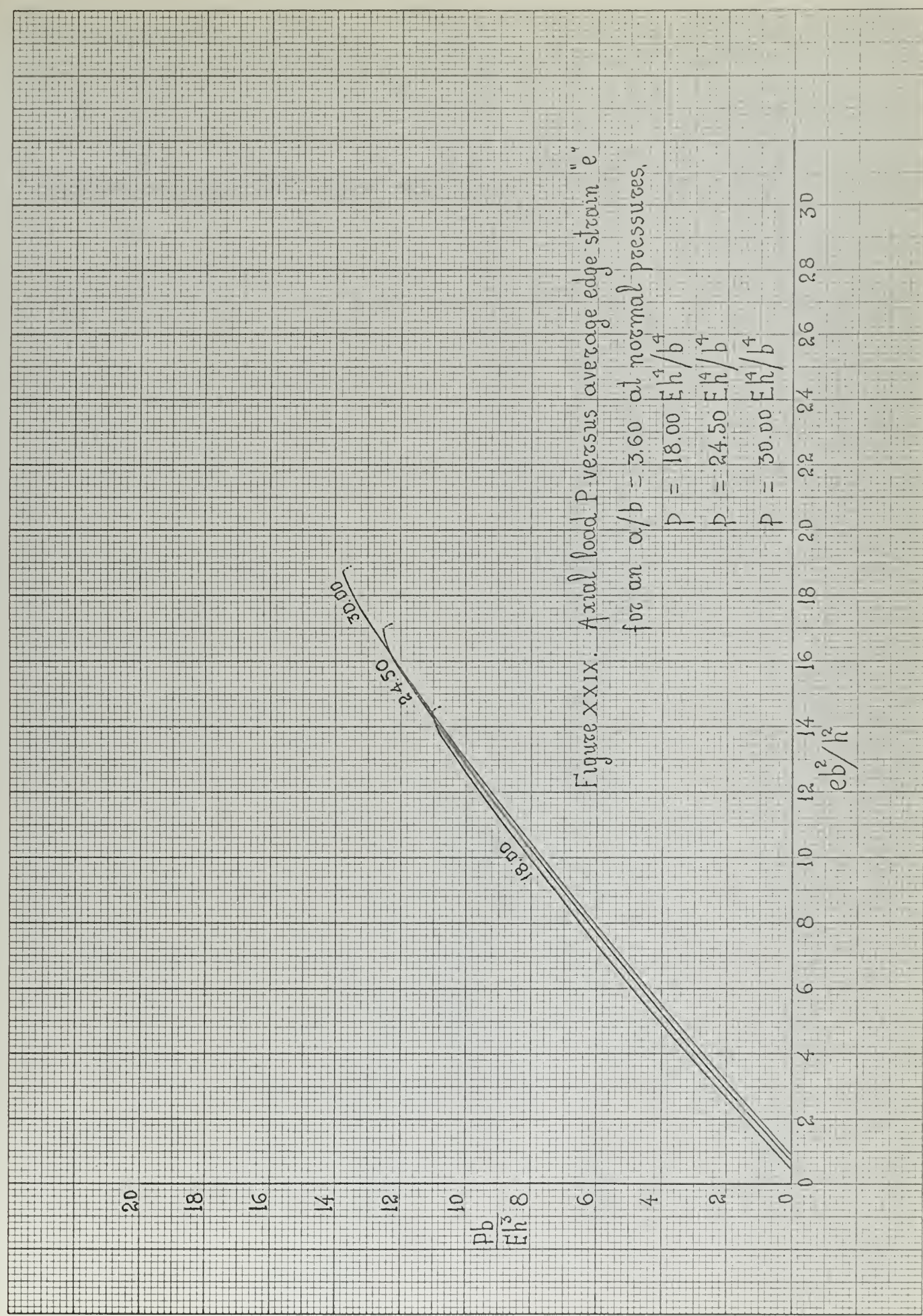
















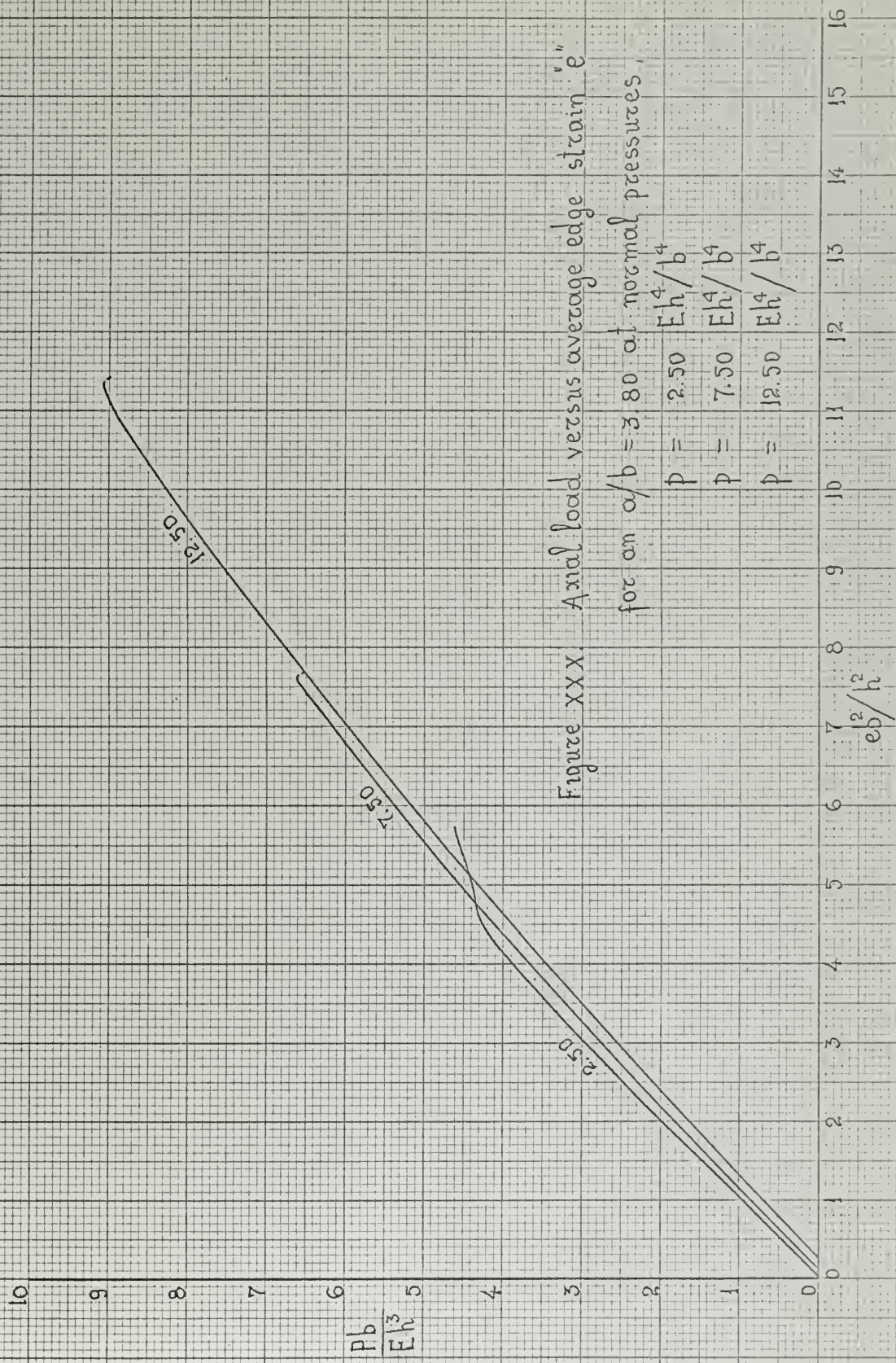


Figure XXX. Axial load versus average edge strain "e"

for an  $a/b = 3.80$  at normal pressures

- $p = 2.50 \frac{Eh^4}{b^4}$
- $p = 7.50 \frac{Eh^4}{b^4}$
- $p = 12.50 \frac{Eh^4}{b^4}$





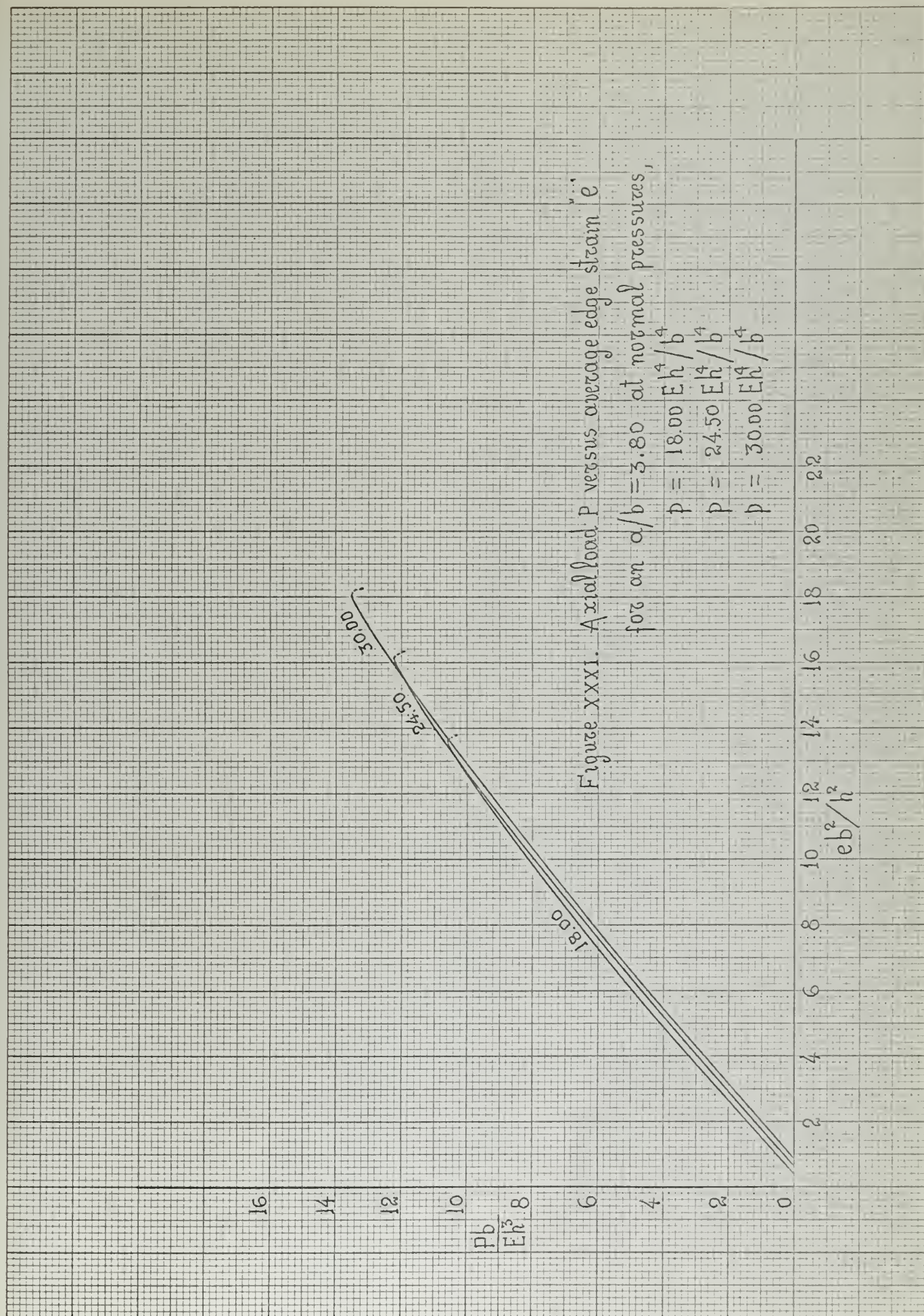


Figure XXXI. Axial load  $P$  versus average edge strain  $e$  for an  $a/b = 3.80$  at normal pressures,

$$p = 18.00 \quad Eh^4/b^4$$

$$p = 24.50 \quad Eh^4/b^4$$

$$p = 30.00 \quad Eh^4/b^4$$

$\frac{Pb}{Eh^3}$

$\frac{eb^2}{h^2}$





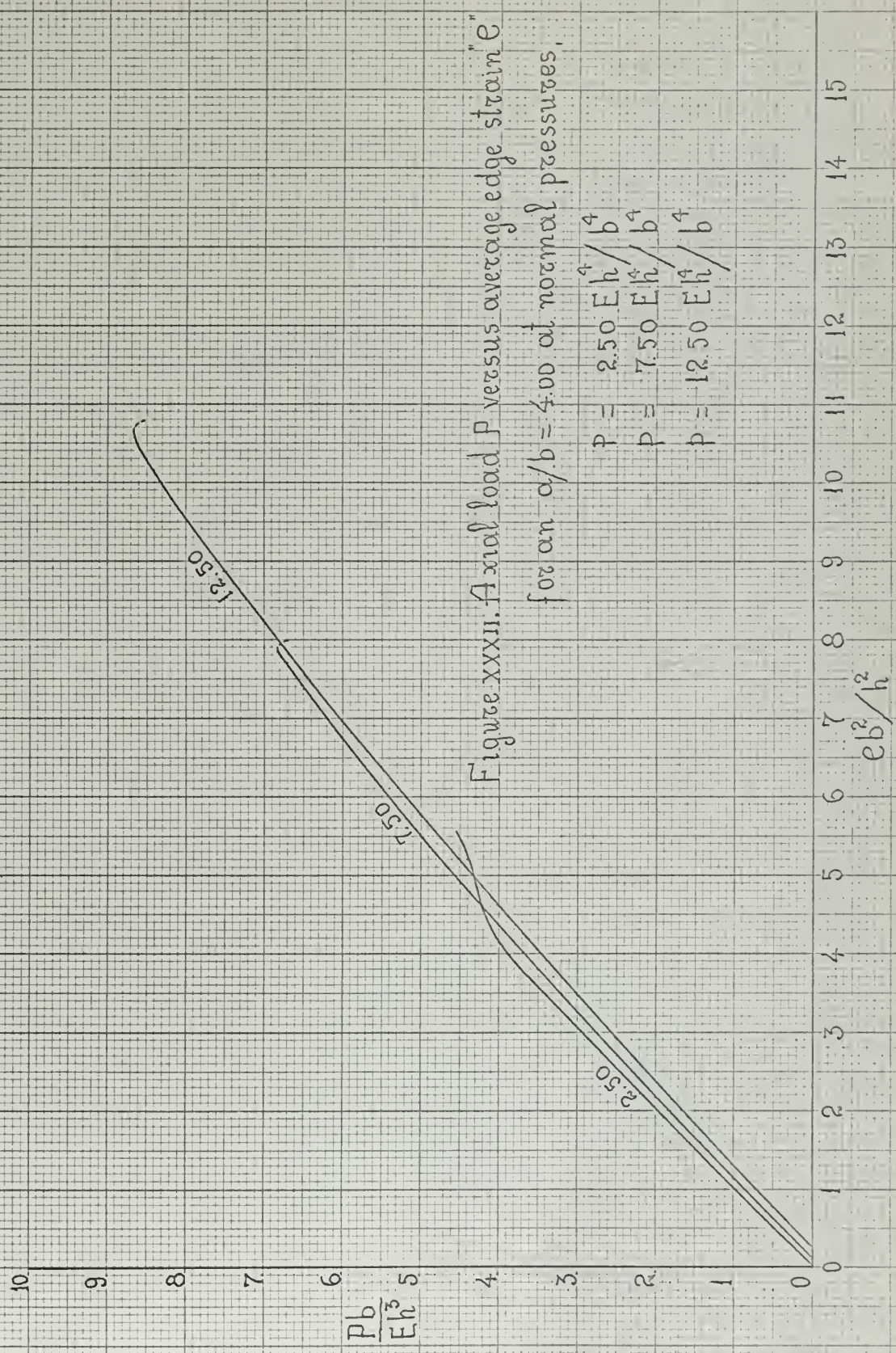


Figure xxxii. Axial load  $P$  versus average edge strain  $e$  for an  $a/b = 4.00$  at normal pressures.





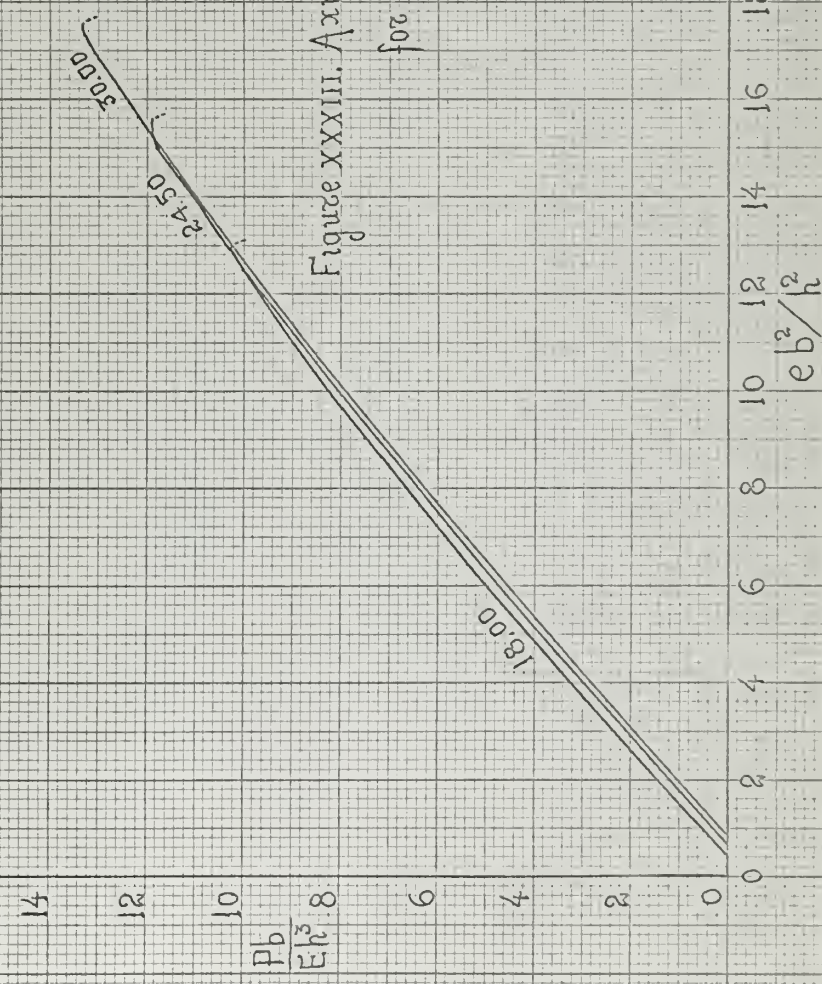


Figure xxxiii. Axial load  $P$  versus average edge strain " $e$ " for an  $a/b = 4.00$  at normal pressures,

- $p = 18.00 \cdot Eh^4/b^4$
- $p = 24.50 \cdot Eh^4/b^4$
- $p = 30.00 \cdot Eh^4/b^4$





Table XIII - Values of  $(Pb/Eh^3)$  critical at various values of  $a/b$  and  $pb^4/Eh^4$ . These are results using equation (8).

a/b	$pb^4/Eh^4 =$					
	2.50	7.50	12.50	18.00	24.50	30.00
0.60	79.51	79.60	79.68	79.78	79.89	79.98
1.00	27.56	28.07	28.59	29.17	29.86	30.46
1.20	16.89	17.49	18.06	18.66	19.32	19.86
1.40	11.33	12.29	13.13	13.95	14.84	15.54
1.60	7.90	9.35	10.48	11.57	12.74	13.67
1.70	6.77	8.41	9.68	10.93	12.30	13.43
1.80	5.93	7.71	9.14	10.60	12.31	13.86
1.90	5.36	7.19	8.82	10.62	13.20	19.35
2.00	4.96	6.83	8.72	11.38	17.03	17.65
2.10	4.69	6.54	8.96	14.79	15.68	16.38

Table XII - Values of  $(P_0/E_0)^3$  critical at various values of  $a/b$  and  $P_0/E_0$ . These are results using equation (II).

$a/b$	$P_0/E_0$					
	2.50	7.50	12.50	18.00	24.50	30.00
0.80	79.51	79.80	79.68	79.78	79.89	79.98
1.00	27.55	28.07	28.35	29.17	29.85	30.48
1.20	16.88	17.45	18.08	18.65	19.35	19.86
1.40	11.33	12.29	13.13	13.65	14.84	15.54
1.60	7.90	9.35	10.43	11.57	12.74	13.67
1.70	6.77	8.41	9.68	10.93	12.30	13.43
1.80	5.93	7.71	9.14	10.60	12.31	13.46
1.90	5.36	7.19	8.83	10.63	13.20	13.35
2.00	4.96	6.83	8.72	11.38	13.03	13.65
2.10	4.69	6.54	8.96	11.75	13.68	13.38



Table XIV - Values of  $(\text{Pb}/\text{Eh}^3)_{\text{critical}}$  at various values of  $a/b$  and  $\text{pb}^4/\text{Eh}^4$ . These are results using equation (7).

a/b	$\text{pb}^4/\text{Eh}^4 =$					
	2.50	7.50	12.50	18.00	24.50	30.00
4.00	4.20	6.82	8.68	10.26	11.98	13.34
3.95	4.16	6.78	8.77	10.33	12.03	13.37
3.90	4.16	6.72	8.87	10.41	12.08	13.40
3.85	4.20	6.67	8.97	10.50	12.14	13.45
3.80	4.30	6.61	9.08	10.58	12.21	13.50
3.60	4.12	6.49	9.64	11.04	12.57	13.77
3.50	4.03	6.49	10.00	11.32	12.80	13.96
3.40	3.98	6.53	10.00	11.65	13.06	14.19
3.20	3.92	6.71	9.13	12.50	13.74	14.77
3.10	3.92	6.86	8.98	13.09	14.17	15.13
3.08	3.92	6.89	8.97	12.47	14.26	15.21
3.04	3.92	6.97	8.96	11.69	14.47	15.38
3.00	3.91	7.05	8.96	11.42	14.70	15.57
2.95	3.91	7.17	8.98	11.19	15.03	15.82
2.90	3.92	7.30	9.03	11.05	13.98	16.11
2.85	3.94	7.44	9.10	10.97	13.40	16.47
2.80	3.99	7.60	9.19	10.93	13.08	15.19
2.60	4.02	8.44	9.79	11.17	12.71	13.98
2.57	4.04	6.89	9.91	11.25	12.74	13.95
2.50	4.10	6.50	10.24	11.50	12.87	13.97
2.40	4.20	6.33	10.83	11.97	13.20	14.18
2.30	4.30	6.31	11.58	12.60	13.72	14.61
2.25	4.36	6.33	9.15	12.99	14.06	14.90
2.20	4.48	6.39	8.81	13.44	14.45	15.26
2.15	4.53	6.47	8.64	11.64	14.91	15.67
2.10	4.69	6.57	8.56	10.93	15.43	16.16
2.05	4.82	6.70	8.56	10.61	13.12	--
2.00	4.97	6.86	8.61	10.45	12.55	14.30
1.90	--	7.27	8.69	10.45	12.15	13.48
1.70	6.87	8.68	10.06	11.36	12.71	13.74



## V. DISCUSSION OF RESULTS

The most remarkable part of the results was the way the critical buckling load behaved with respect to varying values of  $a/b$  (Fig. XXXIV). As can be seen in the corrolary figure (page 64) to Fig. XXXIV, there is a striking resemblance to Bryan's buckling load solution of rectangular plates solely under the action of edge loadings. It may be recalled that the wave-like pattern of Bryan's curve of  $K$  versus  $a/b$  was due to the nature of the  $K$ -formula and to the minimizing procedure adopted in obtaining the critical load for any given value of  $a/b$ . Bryan's  $K$ -formula is a function of the plate's dimensions and of the number of half-sine waves the plate takes after buckling. Thus, it would seem from the foregoing statements to presuppose that the patterns obtained in this thesis are also due in part to the number of half-sine waves.

At first glance the results would give no indication as to the number of waves the plate takes at the incipience of buckling. But, checks had been made by the authors on the relationships between deflections at the midwidth of the plate and at various distances along its length. In other words plots were made of the deflection ratio,  $w/h$ , versus the distance ratio,  $x/a$ , at  $y = b/2$ . These were done for different  $(a/b)$ 's at the same normal pressure. For example, it was found that for a  $p = 18.00 E h^4 / b^4$  there were five buckles at  $a/b = 2.40$ . (The two half-sine waves resulting from the initial general downward deflection of the plate due to normal pressure were not included in the count.) When these checks were expanded to a wide range of values of  $a/b$ , it was found that points lying on the same trough of a  $p$ -curve gave the same number of buckles.

Since only representative tables are given (Tables I to XII) it was deemed unnecessary to insert all these checks in this thesis. If verifications are to be made on this particular point the authors suggest going through the programs shown in Appendix B.



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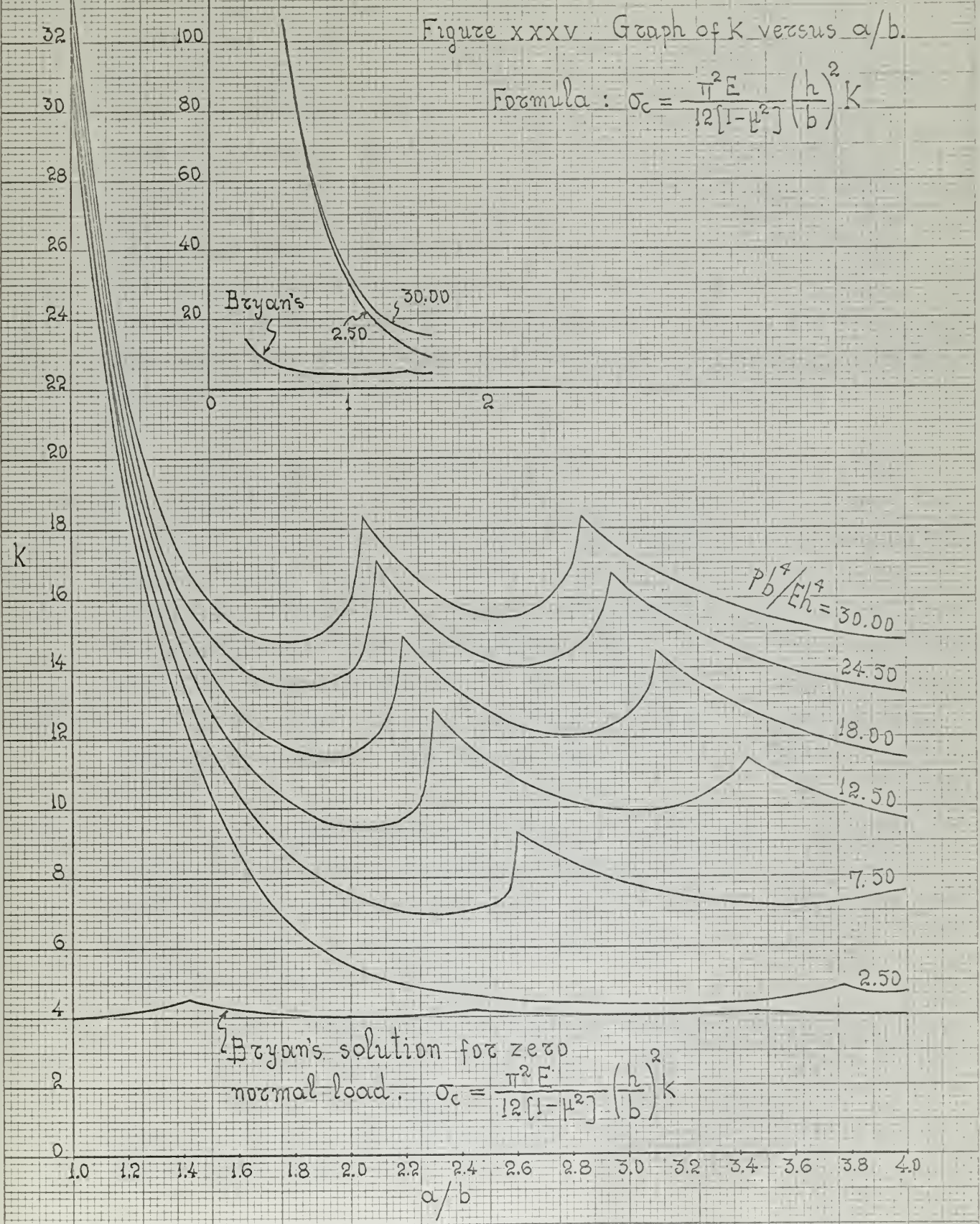
At first glance the results would give no indication as to the number of waves the plate takes at the incipience of buckling. But, checks had been made by the authors on the relationships between deflections at the midwidth of the plate and at various distances along its length. In other words plots were made of the deflection ratio,  $w/h$ , versus the distance ratio,  $x/a$ , at  $y = b/2$ . These were done for different  $a/b$ 's at the same normal pressure. For example, it was found that for a  $p = 18.00 \text{ Eh}^4/b^4$  there were five buckles at  $a/b = 2.40$ . (The two half-sine waves resulting from the initial general downward deflection of the plate due to normal pressure were not included in the count.) When these checks were expanded to a wide range of values of  $a/b$ , it was found that points lying on the same trough of a  $p$ -curve gave the same number of buckles.

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Figure xxxv. Graph of K versus a/b.

Formula:  $\sigma_c = \frac{\pi^2 E}{12[1-\mu^2]} \left(\frac{h}{b}\right)^2 K$







There is, however, no positive explanation as to why only an odd number of buckles is possible. There are either one, three, five, etc., buckles. An even number of buckles, which are obtainable in the case of Bryan's, seems entirely out of the question. On this particular aspect, a possible explanation that the authors believed could have happened was in the manner the deflection equations were obtained. The approximated deflection equations were follow-ups of those used in the references [2], [3], [4]. The equations only have the odd-numbered, subscripted deflection coefficients which, in the final analysis, resulted in an odd function for the  $w$ -equation. At this point, there is no way of predicting what the results will be if the even-numbered subscripts of the deflection coefficients are also taken into consideration.

In reference to the reliability of the results, it is to be noted that the basic theory is valid only within the range of Hooke's law. No allowance was made in the original differential equations on the change of Young's modulus when the axial load exceeds the proportional limit. Hence, all the results of this thesis presupposed that the material had been within the elastic region at all times. Experimental results are not available on the case of plates with the same loading and boundary conditions such that, definite identifications cannot be made as to whether the results are within the elastic region or not.

When a further study is made of Fig. XXXIV for  $a/b$  less than or equal to one, there is a very wide discrepancy between the calculated critical loads and Bryan's that it is quite doubtful whether the expected critical loads are below the yield strength of steel for a majority of plate characteristics.

The following computed results are hoped to illustrate the point:

Consider a plate with a  $b/h = 104.7$  and acted on by a pressure,  $p = 12.50 Eh^4/b^4 = 3.11$  psi. The  $b/h$  may correspond to a 14.02-pound plate with  $b = 36$  inches, and the  $p$  may correspond to a hydrostatic head of 7 feet.

There is, however, no positive explanation as to why only an odd number of buckles is possible. There are either one, three, five, etc., buckles. An even number of buckles, which are obtainable in the case of Bryan's, seems entirely out of the question. On this particular aspect, a possible explanation that the authors believed could have happened was in the manner the deflection equations were obtained. The approximated deflection equations were follow-ups of those used in the references [2], [3], [4]. The equations only have the odd-numbered, subscripted deflection coefficients which, in the final analysis, resulted in an odd function for the w-equation. At this point, there is no way of predicting what the results will be if the even-numbered subscripts of the deflection coefficients are also taken into consideration.

In reference to the reliability of the results, it is to be noted that the basic theory is valid only within the range of Hooke's law. No allowance was made in the original differential equations on the change of Young's modulus when the axial load exceeds the proportional limit. Hence, all the results of this thesis presupposed that the material had been within the elastic region at all times. Experimental results are not available on the case of plates with the same loading and boundary conditions such that, definite identifications cannot be made as to whether the results are within the elastic region or not.

When a further study is made of Fig. XXIV for  $\lambda/b$  less than or equal to one, there is a very wide discrepancy between the calculated critical loads and Bryan's that it is quite doubtful whether the expected critical loads are below the yield strength of steel for a majority of plate characteristics.

The following computed results are hoped to illustrate the point: Consider a plate with a width  $b = 100.7$  and acted on by a pressure,  $p = 12.0 \text{ lb/in}^2$ ,  $b^2 = 10144.49$ . The  $\lambda/b$  may correspond to a  $\lambda = 14.02$  pound plate with  $b = 30$  inches, and the  $p$  may correspond to a hydrostatic head of 7 feet.



Case 1	Case 2	Case 3
$a/b = 1.50$	$a/b = 1.00$	$a/b = 0.60$
$K \approx 12.8$	$K \approx 28$	$K = 78$
$\sigma_c = 31800 \text{ psi}$	$\sigma_c = 69000 \text{ psi}$	$\sigma_c = 192000 \text{ psi}$

If mild steel or high-tensile steel (HTS) were used with yield stresses of 33000 and 47000 psi, respectively, it is apparent that cases 2 and 3 exceed their corresponding yield strengths. Hence, the theory fails for these cases. If HY80 or HY100 were used then theory is found to fail only for case 3. At any rate, the fact remains that the basic theory does not differentiate between the elastic and inelastic regions. A consequent conclusion, therefore, is the need for a check on the yield strength whenever the formulated  $\sigma_c$  of this thesis is used.

Except for the restrictions mentioned in the previous paragraph, it is obvious that the results obtained confirmed the conclusion of Levy on the effect of normal pressure on the buckling load. But, it is also evident that to neglect the normal pressure in predicting buckling strength is unwise design from an economic point of view.

Lastly, the computer programs are limited in scope, although they worked properly in providing all the needed results of this thesis. They were the consequent developments of the approximated deflections equations. A more desirable program would be to start with the equations shown in reference [ 2 ].



Case 1

$$a/b = 1.50$$

$$K = 12.8$$

$$\sigma_c = 37800 \text{ psi}$$

Case 2

$$a/b = 1.00$$

$$K = 20$$

$$\sigma_c = 33000 \text{ psi}$$

Case 3

$$a/b = 0.50$$

$$K = 72$$

$$\sigma_c = 108000 \text{ psi}$$

If mild steel or high-tensile steel (HTS) were used with yield stresses of 33000 and 17000 psi, respectively, it is apparent that cases 2 and 3 exceed their corresponding yield strengths. Hence, the theory fails for these cases. If HY80 or HY100 were used then theory is found to fail only for case 3. At any rate, the fact remains that the basic theory does not differentiate between the elastic and inelastic regions. A consequent conclusion, therefore, is the need for a check on the yield stress whenever the formulated  $\sigma_c$  of this theory is used.

Except for the restrictions mentioned in the previous paragraph, it is obvious that the results obtained confirmed the conclusion of Levy on the effect of normal pressure on the buckling load. Evidently, it is also evident that to neglect the normal pressure in predicting buckling strength is unwise design from an economic point of view.

Lastly, the computer programs are limited in scope. Although they worked properly in providing all the needed results of this thesis. They were the consequent developments of the approximated deflections equations. A more feasible program would be to start with the equations shown in reference [3].

## VI. CONCLUSIONS AND RECOMMENDATIONS

### Conclusions

1. Normal pressure always increases the buckling strength of rectangular flat plates. However, the buckling loads obtained by means of Levy's solution are not conclusive values; that is, the buckling load may or may not be within the proportional limit. In other words, theory does not differentiate between the elastic and inelastic regions.

2. Plotted values of critical loads against the ratio,  $a/b$ , for a constant value of normal pressure exhibits the same trend and characteristics as the plot of Bryan's formula for zero pressure.

3. To neglect the effect of normal pressure in the determination of buckling load is unwise design from an economic standpoint.

4. In general, the buckling load formula can be written in the form,

$$\sigma_c = \frac{\pi^2 E}{12(1-\mu^2)} \left(\frac{h}{b}\right)^2 K$$

where, this time,  $K$  is not only a function of plate geometry but also a function of the normal pressure and Poisson's ratio as well.

### Recommendations

1. A better representative plot of  $K$  versus  $a/b$  is needed by including the even-numbered subscripted deflection coefficients of Levy's solution.

2. In consonance with the first recommendation, a revised computer program is desirable, possibly to start right at the series

## VI. CONCLUSIONS AND RECOMMENDATIONS

### Conclusions

1. Normal pressure always increases the buckling strength of cylindrical flat plates. However, the buckling loads obtained by means of Levy's solution are not conclusive values; that is, the buckling formulae are only valid within the proportional limit. In other words, there is not direct relationship between the elastic and inelastic regions.

2. Plastic values of critical load is against the value,  $\sigma_c$ , for a constant value of normal pressure which the same yield and characteristic of the plot of Bryan's formula for zero pressure.

3. To neglect the effect of normal pressure in the determination of buckling load is not desirable from an economic standpoint.

4. In general, the buckling load formula can be written in the

form,

$$\sigma_c = \frac{E}{(1-\nu^2)} \left( \frac{b}{d} \right)^2 K$$

where, this time,  $K$  is not only a function of plate geometry but also a function of the normal pressure and Poisson's ratio as well.

### Recommendations

1. A better representative plot of  $K$  versus  $\nu/d$  is needed by including the even-numbered and odd-numbered deflection coefficients of Levy's solution.

2. In accordance with the 1st recommendation, a revised constant  $K$  can be established, provided that the same as the series



forms of the equations shown in reference [ 2]. This step would eliminate using approximate deflection equations.

3. It is recommended that a study be made of the most feasible experimental set-up to simulate the boundary conditions stated in this thesis; and hence, the ensuing experimental verifications.

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3. It is recommended that a study be made of the most feasible experimental set-up to simulate the boundary conditions stated in this thesis and hence, the ensuing experimental verification.

## APPENDIX A

### DEFLECTION EQUATIONS

The six (6) simultaneous cubic equations involved in the solution of the deflection equation,

$$\begin{aligned}
 w = & w_{1,1} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} + w_{1,3} \sin \frac{\pi x}{a} \sin \frac{3\pi y}{b} \\
 & + w_{3,1} \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b} + w_{3,3} \sin \frac{3\pi x}{a} \sin \frac{3\pi y}{b} \\
 & + w_{1,5} \sin \frac{\pi x}{a} \sin \frac{5\pi y}{b} + w_{5,1} \sin \frac{5\pi x}{a} \sin \frac{\pi y}{b}
 \end{aligned}$$

are as follows\*:

$$\begin{aligned}
 0 = & - \frac{256 a_1}{\pi^6} \frac{pb^4}{Eh} + \frac{4}{3(1-\mu^2)} \frac{h^2}{a_3} w_{1,1} - \frac{16}{\pi^6} \frac{Pb}{Eh} w_{1,1} \\
 & + (a_1 + a_2) w_{1,1}^3 - 3a_2 w_{1,1}^2 w_{1,3} - 3a_1 w_{1,1}^2 w_{3,1} \\
 & + (9a_1 + 4a_2 + 16a_3 + a_8) w_{1,1} w_{1,3}^2 + (4a_1 + 9a_2 + 16a_3 \\
 & + a_4) w_{1,1} w_{3,1}^2 + 32a_3 w_{1,1} w_{1,3} w_{3,1} + 81(a_4 + a_8) w_{1,1} w_{3,3}^2 \\
 & - 18(a_1 + a_8) w_{1,1} w_{1,3} w_{3,3} - 18(a_2 + a_4) w_{1,1} w_{3,1} w_{3,3} \\
 & - (9a_1 + 64a_3 + 25a_8) w_{1,3}^2 w_{3,1} - (9a_2 + 64a_3 + 25a_4) w_{1,3} w_{3,1}^2 \\
 & + (36a_1 + 36a_2 + 225a_4 + 225a_8) w_{1,3} w_{3,1} w_{3,3} - 6(a_2 \\
 & + 3a_8) w_{1,1} w_{1,3} w_{1,5} - 6(a_1 + 3a_4) w_{1,1} w_{3,1} w_{5,1} \\
 & + 162a_8 w_{1,1} w_{3,3} w_{1,5} + 162a_4 w_{1,1} w_{3,3} w_{5,1} + (25a_1 + 4a_2
 \end{aligned}$$

\*The corresponding significance of each subscripted  $a$  is shown at the end of this Appendix.



APPENDIX A  
DEFLECTION EQUATIONS

The six (6) simultaneous cubic equations involved in the solution of the deflection equation,

$$\begin{aligned} &w_{1,1} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} + w_{1,2} \sin \frac{\pi x}{a} \sin \frac{2\pi y}{b} + w_{1,3} \sin \frac{\pi x}{a} \sin \frac{3\pi y}{b} \\ &+ w_{2,1} \sin \frac{2\pi x}{a} \sin \frac{\pi y}{b} + w_{2,2} \sin \frac{2\pi x}{a} \sin \frac{2\pi y}{b} + w_{2,3} \sin \frac{2\pi x}{a} \sin \frac{3\pi y}{b} \\ &+ w_{3,1} \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b} + w_{3,2} \sin \frac{3\pi x}{a} \sin \frac{2\pi y}{b} + w_{3,3} \sin \frac{3\pi x}{a} \sin \frac{3\pi y}{b} \end{aligned}$$

are as follows\*:

$$\begin{aligned} 0 = & \frac{\pi^4}{6} \frac{D^4}{Eh^3} + \frac{1}{2(1-\nu^2)} \frac{h^2}{8} w_{1,1}^2 - \frac{16}{9} \frac{D^2}{Eh^3} w_{1,1}^2 \\ & + (a_1 + a_2) w_{1,1}^3 - 3a_2 w_{1,1}^2 w_{1,2} - 3a_1 w_{1,1} w_{1,2}^2 + (a_1 + a_2 + 10a_3 + 10a_4) w_{1,1}^2 w_{1,2} \\ & + (2a_1 + 2a_2 + 3a_3 + 3a_4) w_{1,1} w_{1,2}^2 + 3a_2 w_{1,1}^2 w_{1,3} + 3a_1 w_{1,1} w_{1,3}^2 + 3a_3 w_{1,1}^2 w_{1,3} \\ & + 18(a_1 + a_2) w_{1,1} w_{1,2} w_{1,3} - 18(a_2 + a_4) w_{1,1}^2 w_{1,3}^2 + 3a_1 w_{1,1}^2 w_{1,3}^2 \\ & - (2a_1 + 2a_2 + 3a_3 + 3a_4) w_{1,1}^2 w_{1,3}^2 + 3a_2 w_{1,1}^2 w_{1,3}^2 + 3a_1 w_{1,1}^2 w_{1,3}^2 \\ & + (3a_1 + 3a_2 + 3a_3 + 3a_4) w_{1,1}^2 w_{1,3}^2 + 3a_2 w_{1,1}^2 w_{1,3}^2 + 3a_1 w_{1,1}^2 w_{1,3}^2 \\ & + 3a_2 w_{1,1}^2 w_{1,3}^2 - 3(a_1 + a_2) w_{1,1}^2 w_{1,3}^2 + 3a_2 w_{1,1}^2 w_{1,3}^2 + 3a_1 w_{1,1}^2 w_{1,3}^2 \\ & + 18a_2 w_{1,1}^2 w_{1,3}^2 + 18a_1 w_{1,1}^2 w_{1,3}^2 + 18a_2 w_{1,1}^2 w_{1,3}^2 + 18a_1 w_{1,1}^2 w_{1,3}^2 \end{aligned}$$

\*The corresponding significance of each subscript is shown at the end of this Appendix.

$$\begin{aligned}
& + 81 a_8 + 16 a_9) w_{1,1} w_{1,5}^2 + (3a_2 + 64a_3) w_{1,3}^2 w_{1,5} \\
& + (3a_1 + 64a_3) w_{3,1}^2 w_{5,1} + (64a_3 + 274a_8) w_{1,3} w_{3,1} w_{1,5} \\
& + (64a_3 + 274a_4) w_{1,3} w_{3,1} w_{5,1} - 144 (a_3 + a_9) w_{1,3} w_{3,3} w_{1,5} \\
& - 9(a_1 + 16a_3 + 49a_4 + 9a_8) w_{1,3} w_{3,3} w_{5,1} - 9(a_2 + 16a_3 \\
& + 9a_4 + 49a_8) w_{3,1} w_{3,3} w_{1,5} - 144(a_3 + a_5) w_{3,1} w_{3,3} w_{5,1} \\
& + 9(a_2 + 81a_4) w_{3,3}^2 w_{1,5} + 9(a_1 + 81a_8) w_{3,3}^2 w_{5,1} \\
& - (25a_2 + 441a_4 + 256a_5) w_{1,3} w_{5,1}^2 - (25a_1 + 441a_8 \\
& + 256a_9) w_{3,1} w_{1,5}^2 + 729(a_4 + a_8) w_{3,3} w_{1,5} w_{5,1} \cdot \\
0 = & - \frac{256a_1}{3\pi^6} \frac{pb^4}{Fh} + \frac{4}{3(1-\mu^2)} \frac{h^2}{a_9} w_{1,3} - \frac{16}{\pi} \frac{Pb}{Fh} w_{1,3} - a_2 w_{1,1}^3 \\
& + (9a_1 + 4a_2 + 16a_3 + a_8) w_{1,1}^2 w_{1,3} + 16a_3 w_{1,1}^2 w_{3,1} \\
& - 9(a_1 + a_8) w_{1,1}^2 w_{3,3} - (9a_2 + 64a_3 + 25a_4) w_{1,1} w_{3,1}^2 \\
& - 2(9a_1 + 64a_3 + 25a_8) w_{1,1} w_{1,3} w_{3,1} + (36a_1 + 36a_2 \\
& + 225a_4 + 225a_8) w_{1,1} w_{3,1} w_{3,3} + (81a_1 + a_2) w_{1,3}^3 \\
& + (272a_3 + 625a_4 + 625a_8) w_{1,3} w_{3,1}^2 - 243a_1 w_{1,3}^2 w_{3,3} \\
& + 9(36a_1 + a_2 + 144a_9 + 9a_{13}) w_{1,3} w_{3,3}^2 - 3(a_2 + 3a_8) w_{1,1}^2 w_{1,5} \\
& + 2(3a_2 + 64a_3) w_{1,1} w_{1,3} w_{1,5} + (64a_3 + 274a_8) w_{1,1} w_{3,1} w_{1,5} \\
& + (64a_3 + 274a_4) w_{1,1} w_{3,1} w_{5,1} - 144(a_3 + a_9) w_{1,1} w_{3,3} w_{1,5} \\
& - 9(a_1 + 16a_3 + 49a_4 + 9a_8) w_{1,1} w_{3,3} w_{5,1} - (25a_2 + 441a_4 \\
& + 256a_5) w_{1,1} w_{5,1}^2 - (9a_2 + 64a_3 + 1225a_8) w_{3,1}^2 w_{1,5} \\
& - 256a_3 w_{3,1}^2 w_{5,1} - 512a_3 w_{1,3} w_{3,1} w_{1,5} - 2(9a_1 + 64a_3 \\
& + 1225a_4) w_{1,3} w_{3,1} w_{5,1} + 9(2a_2 + 64a_3 + 225a_4
\end{aligned}$$

[illegible]



$$\begin{aligned}
& + 256a_9 + 49a_{13}) w_{3,1} w_{3,3} w_{1,5} + 9(2a_1 + 64a_3 + 256a_5 \\
& + 225a_8 + 49a_{12}) w_{3,1} w_{3,3} w_{5,1} + (225a_1 + 4a_2 + 256a_3 \\
& + a_{10}) w_{1,3} w_{1,5}^2 + (256a_3 + 2401a_4 + 4096a_5 \\
& + 2401a_{12}) w_{1,3} w_{5,1}^2 + 656a_3 w_{3,1} w_{1,5} w_{5,1} - 9(25a_1 \\
& + 64a_3 + 9a_{10}) w_{3,3} w_{1,5}^2 - 9(64a_3 + 144a_4 + 169a_{13}) w_{3,3} w_{1,5} w_{5,1} \\
& - (25a_2 + 576a_3 + 8281a_{12}) w_{1,5} w_{5,1}^2 \\
0 = & - \frac{256a_1}{3\pi^6} \frac{pb^4}{Eh} + \frac{4}{3(1-\mu^2)} \frac{h^2}{a_5} w_{3,1} - \frac{144}{\pi^2} \frac{Pb}{Eh} w_{3,1} - a_1 w_{1,1}^3 \\
& + 16a_3 w_{1,1}^2 w_{1,3} + (4a_1 + 9a_2 + 16a_3 + a_4) w_{1,1}^2 w_{3,1} - 9(a_2 \\
& + a_4) w_{1,1}^2 w_{3,3} - (9a_1 + 64a_3 + 25a_8) w_{1,1} w_{1,3}^2 - 2(9a_2 \\
& + 64a_3 + 25a_4) w_{1,1} w_{1,3} w_{3,1} + (36a_1 + 36a_2 + 225a_4 \\
& + 225a_8) w_{1,1} w_{1,3} w_{3,3} + (272a_3 + 625a_4 + 625a_8) w_{1,3}^2 w_{3,1} \\
& + (a_1 + 81a_2) w_{3,1}^3 - 243a_2 w_{3,1}^2 w_{3,3} + 9(a_1 + 36a_2 + 144a_5 \\
& + 9a_{12}) w_{3,1} w_{3,3}^2 - 3(a_1 + 3a_4) w_{1,1}^2 w_{5,1} + (64a_3 \\
& + 274a_8) w_{1,1} w_{1,3} w_{5,1} + (64a_3 + 274a_4) w_{1,1} w_{1,3} w_{5,1} \\
& + 2(3a_1 + 64a_3) w_{1,1} w_{3,1} w_{5,1} - 9(2a_2 + 16a_3 + 9a_4 \\
& + 49a_8) w_{1,1} w_{3,3} w_{1,5} - 144(a_3 + a_5) w_{1,1} w_{3,3} w_{5,1} \\
& - (25a_1 + 441a_8 + 256a_9) w_{1,1} w_{1,5}^2 - 256a_3 w_{1,3}^2 w_{1,5} \\
& - (9a_1 + 64a_3 + 1225a_4) w_{1,3}^2 w_{5,1} - 2(9a_2 + 64a_3 \\
& + 1225a_8) w_{1,3} w_{3,1} w_{1,5} - 512a_3 w_{1,3} w_{3,1} w_{5,1} + 9(2a_2 \\
& + 64a_3 + 225a_4 + 256a_9 + 49a_{13}) w_{1,3} w_{3,3} w_{1,5} \\
& + 9(2a_1 + 64a_3 + 256a_5 + 225a_8 + 49a_{12}) w_{1,3} w_{3,3} w_{5,1}
\end{aligned}$$

[illegible]

$$\begin{aligned}
& + (256a_3 + 2401a_8 + 4096a_9 + 2401a_{13}) w_{3,1} w_{1,5}^2 \\
& + (4a_1 + 225a_2 + 256a_3 + a_6) w_{3,1} w_{5,1}^2 + 656a_3 w_{1,3} w_{1,5} w_{5,1} \\
& - 9(25a_2 + 64a_3 + 9a_6) w_{3,3} w_{5,1}^2 - 9(64a_3 + 441a_8 \\
& + 169a_{12}) w_{3,3} w_{1,5} w_{5,1} - (25a_1 + 576a_3 + 8281a_{13}) w_{1,5}^2 w_{5,1} \\
0 = & - \frac{256a_1}{9\pi^6} \frac{pb^4}{Eh} + \frac{324}{3(1-\mu^2)} \frac{h^2}{a_3} w_{3,3} - \frac{144}{\pi^2} \frac{Pb}{Eh} w_{3,3} \\
& - 9(a_1 + a_8) w_{1,1}^2 w_{1,3} - 9(a_2 + a_4) w_{1,1}^2 w_{3,1} + 81(a_4 + a_8) w_{1,1}^2 w_{3,3} \\
& + (36a_1 + 36a_2 + 225a_4 + 225a_8) w_{1,1} w_{1,3} w_{3,1} - 81a_1 w_{1,3}^3 \\
& - 81a_2 w_{3,1}^3 + 9(36a_1 + a_2 + 144a_9 + 9a_{13}) w_{1,3}^2 w_{3,3} \\
& + 81(a_1 + a_2) w_{3,3}^3 + 9(a_1 + 36a_2 + 144a_5 + 9a_{12}) w_{3,1}^2 w_{3,3} \\
& + 81a_8 w_{1,1}^2 w_{1,5} + 81a_4 w_{1,1}^2 w_{5,1} - 144(a_3 + a_9) w_{1,1} w_{1,3} w_{1,5} \\
& - 9(2a_1 + 16a_3 + 49a_4 + 9a_8) w_{1,1} w_{1,3} w_{5,1} - 9(2a_2 \\
& + 16a_3 + 9a_4 + 49a_8) w_{1,1} w_{3,1} w_{1,5} - 144(a_3 + a_5) w_{1,1} w_{3,1} w_{5,1} \\
& + 18(a_2 + 81a_4) w_{1,1} w_{3,3} w_{1,5} + 18(a_1 + 18a_8) w_{1,1} w_{3,3} w_{5,1} \\
& + 729(a_4 + a_8) w_{1,1} w_{1,5} w_{5,1} + 9(2a_2 + 64a_3 + 225a_4 \\
& + 256a_9 + 49a_{13}) w_{1,3} w_{3,1} w_{1,5} + 9(2a_1 + 64a_3 + 256a_5 \\
& + 225a_8 + 49a_{12}) w_{1,3} w_{3,1} w_{5,1} - 9(25a_1 + 64a_3 \\
& + 9a_{10}) w_{1,3} w_{1,5}^2 - 9(25a_2 + 64a_3 + 9a_6) w_{3,1} w_{5,1}^2 \\
& - 9(64a_3 + 441a_4 + 169a_{13}) w_{1,3} w_{1,5} w_{5,1} - 9(64a_3 \\
& + 441a_8 + 169a_{12}) w_{3,1} w_{1,5} w_{5,1} + 81(16a_3 + 81a_4 \\
& + a_8 + 81a_{10}) w_{3,3} w_{1,5}^2 + 81(16a_3 + a_4 + 81a_6
\end{aligned}$$



[illegible]

$$+ 81 a_8) w_{3,3} w_{5,1}^2 + 2592 a_3 w_{3,3} w_{5,1}.$$

$$\begin{aligned} 0 = & -\frac{256a_1}{5\pi^6} \frac{pb^4}{Eh} + \frac{4}{3(1-\mu^2)} \frac{h^2}{a_{11}} w_{1,5} - \frac{16}{\pi^2} \frac{Pb}{Eh} w_{1,5} \\ & - 3(a_2 + 3a_8) w_{1,1}^2 w_{1,3} + 81a_8 + (3a_2 + 64a_3) w_{1,1} w_{1,3}^2 \\ & + (64a_3 + 274a_8) w_{1,1} w_{1,3} w_{3,1} + 9(a_2 + 81a_4) w_{1,1} w_{3,3}^2 \\ & - 144(a_3 + a_9) w_{1,1} w_{1,3} w_{3,3} - 9(2a_2 + 16a_3 + 9a_4 \\ & + 49a_8) w_{1,1} w_{3,1} w_{3,3} - 256a_3 w_{1,3}^2 w_{3,1} - (9a_2 + 64a_3 \\ & + 1225a_8) w_{1,3} w_{3,1}^2 + 9(2a_2 + 64a_3 + 225a_4 + 256a_9 \\ & + 49a_{13}) w_{1,3} w_{3,1} w_{3,3} + (25a_1 + 4a_2 + 81a_8 + 16a_9) w_{1,1}^2 w_{1,5} \\ & - 2(25a_1 + 441a_8 + 256a_9) w_{1,1} w_{3,1} w_{1,5} + 729(a_4 \\ & + a_8) w_{1,1} w_{3,3} w_{5,1} + (225a_1 + 4a_2 + 256a_3 + a_{10}) w_{1,3}^2 w_{1,5} \\ & + (256a_3 + 2401a_8 + 4096a_9 + 2401a_{13}) w_{3,1}^2 w_{1,5} \\ & + 656a_3 w_{1,3} w_{3,1} w_{5,1} - 18(25a_1 + 64a_3 + 9a_{10}) w_{1,3} w_{3,3} w_{1,5} \\ & - 9(64a_3 + 441a_4 + 169a_{13}) w_{1,3} w_{3,3} w_{5,1} - 9(64a_3 + 441a_8 \\ & + 169a_{12}) w_{3,1} w_{3,3} w_{5,1} + 81(16a_3 + 81a_4 + a_8 + 81a_{10}) w_{3,3}^2 w_{1,5} \\ & + 1296a_3 w_{3,3}^2 w_{5,1} - (25a_2 + 576a_3 + 8281a_{12}) w_{1,3} w_{5,1}^2 \\ & - 2(25a_1 + 576a_3 + 8281a_{13}) w_{3,1} w_{1,5} w_{5,1} + (625a_1 + a_2) w_{1,5}^3 \\ & + (1552a_3 + 28561a_{12} + 28561a_{13}) w_{1,5} w_{5,1}^2. \end{aligned}$$

$$\begin{aligned} 0 = & -\frac{256a_1}{5\pi^6} \frac{pb^4}{Eh} + \frac{4}{3(1-\mu^2)} \frac{h^2}{a_7} w_{5,1} - \frac{400}{\pi^2} \frac{Pb}{Eh} w_{5,1} \\ & - 3(a_1 + 3a_4) w_{1,1}^2 w_{3,1} + 81a_4 w_{1,1}^2 w_{3,3} + (3a_1 + 64a_3) w_{1,1} w_{3,1}^2 \end{aligned}$$

$$0 = -\frac{1}{2\pi} \frac{d^2 p}{d\tau^2} + \frac{4}{3(1-\tau^2)} \frac{d p}{d\tau} - \frac{10}{3} \frac{p}{\tau} \frac{d p}{d\tau} + \dots$$



$$\begin{aligned}
& + (64a_3 + 274a_4)w_{1,1}w_{1,3}w_{3,1} + 9(a_1 + 81a_8)w_{1,1}w_{3,3}^2 \\
& - 9(2a_1 + 16a_3 + 49a_4 + 9a_8)w_{1,1}w_{1,3}w_{3,3} - 144(a_3 \\
& + a_5)w_{1,1}w_{3,1}w_{3,3} - (9a_1 + 64a_3 + 1225a_4)w_{1,3}^2w_{3,1} \\
& - 256a_3w_{1,3}w_{3,1}^2 + 9(2a_1 + 64a_3 + 256a_5 + 225a_8 \\
& + 49a_{12})w_{1,3}w_{3,1}w_{3,3} + (4a_1 + 25a_2 + 81a_4 + 16a_5)w_{1,1}^2w_{5,1} \\
& - 2(25a_2 + 441a_4 + 256a_5)w_{1,1}w_{1,3}w_{5,1} + 729(a_4 \\
& + a_8)w_{1,1}w_{3,3}w_{1,5} + (256a_3 + 2401a_4 + 4096a_5 + 2401a_{12})w_{1,3}^2w_{5,1} \\
& + (4a_1 + 225a_2 + 256a_3 + a_6)w_{3,1}^2w_{5,1} + 656a_3w_{1,3}w_{3,1}w_{1,5} \\
& - 9(64a_3 + 441a_4 + 169a_{13})w_{1,3}w_{3,3}w_{1,5} - 9(64a_3 + 441a_8 \\
& + 169a_{12})w_{3,1}w_{3,3}w_{1,5} - 18(25a_2 + 64a_3 + 9a_6)w_{3,1}w_{3,3}w_{5,1} \\
& + 1296a_3w_{3,3}^2w_{1,5} + 81(16a_3 + a_4 + 81a_6 + 81a_8)w_{3,3}^2w_{5,1} \\
& - (25a_1 + 576a_3 + 8281a_{13})w_{3,1}w_{1,5}^2 - 2(25a_2 + 576a_3 \\
& + 8281a_{12})w_{1,3}w_{1,5}w_{5,1} + (1552a_3 + 28561a_{12} + 28561a_{13})w_{1,5}^2w_{5,1} \\
& + (a_1 + 625a_2)w_{5,1}^3.
\end{aligned}$$

The four (4) simultaneous equations involved in the solution of the deflection equation,

$$\begin{aligned}
w = & w_{1,1} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} + w_{3,1} \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b} \\
& + w_{5,1} \sin \frac{5\pi x}{a} \sin \frac{\pi y}{b} + w_{7,1} \sin \frac{7\pi x}{a} \sin \frac{\pi y}{b},
\end{aligned}$$

are as follows:

The four (4) simultaneous equations involved in the solution of the deflection equation,

976 210101

$$\begin{aligned}
0 = & -\frac{256a_1}{\pi^6} \frac{pb^4}{Eh} + \frac{4}{3(1-\mu^2)} \frac{h^2}{a_3} w_{1,1} - \frac{16}{\pi^2} \frac{Pb}{Eh} w_{1,1} + (a_1 + a_2) w_{1,1}^3 \\
& - 3a_1 w_{1,1}^2 w_{3,1} + (4a_1 + 9a_2 + 16a_3 + a_4) w_{1,1} w_{3,1}^2 \\
& + (3a_1 + 64a_3) w_{3,1}^2 w_{5,1} - (3a_1 + 25a_4) w_{3,1}^2 w_{7,1} + (4a_1 \\
& + 25a_2 + 81a_4 + 16a_5) w_{1,1} w_{5,1}^2 + (4a_1 + 49a_2 + 256a_5 \\
& + 81a_6) w_{1,1} w_{7,1}^2 - 6(a_1 + 3a_4) w_{1,1} w_{3,1} w_{5,1} - 2(3a_1 \\
& + 64a_5) w_{1,1} w_{5,1} w_{7,1} + (6a_1 + 144a_3 + 225a_4 \\
& + 9a_6) w_{3,1} w_{5,1} w_{7,1}.
\end{aligned}$$

$$\begin{aligned}
0 = & -\frac{256a_1}{3\pi^6} \frac{pb^4}{Eh} + \frac{4}{3(1-\mu^2)} \frac{h^2}{a_5} w_{3,1} - \frac{144}{\pi^2} \frac{Pb}{Eh} w_{3,1} - a_1 w_{1,1}^3 \\
& + (4a_1 + 9a_2 + 16a_3 + a_4) w_{1,1}^2 w_{3,1} - 3(a_1 + 3a_4) w_{1,1}^2 w_{5,1} \\
& + (a_1 + 81a_2) w_{3,1}^3 + (4a_1 + 225a_2 + 256a_3 + a_6) w_{3,1} w_{5,1}^2 \\
& + 3(a_1 + 192a_3) w_{5,1}^2 w_{7,1} + (4a_1 + 441a_2 + 625a_4 + 16a_7) w_{3,1} w_{7,1}^2 \\
& + 2(3a_1 + 64a_3) w_{1,1} w_{3,1} w_{5,1} - 2(3a_1 + 25a_4) w_{1,1} w_{3,1} w_{7,1} \\
& + 3(2a_1 + 48a_3 + 75a_4 + 3a_6) w_{1,1} w_{5,1} w_{7,1}.
\end{aligned}$$

$$\begin{aligned}
0 = & -\frac{256a_1}{5\pi^6} \frac{pb^4}{Eh} + \frac{4}{3(1-\mu^2)} \frac{h^2}{a_7} w_{5,1} - \frac{400}{\pi^2} \frac{Pb}{Eh} w_{5,1} \\
& - 3(a_1 + 3a_4) w_{1,1}^2 w_{3,1} + (4a_1 + 25a_2 + 81a_4 + 16a_5) w_{1,1}^2 w_{5,1} \\
& - (3a_1 + 64a_5) w_{1,1}^2 w_{7,1} + (3a_1 + 64a_3) w_{1,1} w_{3,1}^2 + (4a_1 + 225a_2 \\
& + 256a_3 + a_6) w_{3,1}^2 w_{5,1} + (a_1 + 625a_2) w_{5,1}^3 + (4a_1 + 1225a_2 \\
& + 1296a_3 + a_4) w_{5,1} w_{7,1}^2 + 3(2a_1 + 48a_3 + 75a_4 + 3a_6) w_{1,1} w_{3,1} w_{7,1} \\
& + 6(a_1 + 192a_3) w_{3,1} w_{5,1} w_{7,1}.
\end{aligned}$$



[illegible]

$$0 = \frac{3w}{8} \frac{dp^{\frac{1}{2}}}{dt} + \frac{H^{\frac{1}{2}}}{S(1-\alpha)} \frac{dw}{dt} - \frac{1}{n} \frac{dK}{dt} w^{\frac{1}{2}} - \frac{1}{\beta} \frac{dP}{dt} w^{\frac{1}{2}}$$

[illegible]

$$\begin{aligned}
0 = & -\frac{256a_1}{7\pi^6} \frac{pb^4}{Eh} + \frac{4}{3(1-\mu^2)} \frac{h^2}{a_{15}} w_{7,1} - \frac{784}{\pi^2} \frac{Pb}{Eh} w_{7,1} \\
& - (3a_1 + 64a_5) w_{1,1}^2 w_{5,1} + (4a_1 + 49a_2 + 256a_5 + 81a_6) w_{3,1}^2 w_{7,1} \\
& - (3a_1 + 25a_4) w_{1,1} w_{3,1}^2 + (4a_1 + 441a_2 + 625a_4 + 16a_7) w_{3,1}^2 w_{7,1} \\
& + 3(a_1 + 192a_3) w_{3,1} w_{5,1}^2 + (4a_1 + 1225a_2 + 1296a_3 + a_{14}) w_{5,1}^2 w_{7,1} \\
& + (a_1 + 2401a_2) w_{7,1}^3 + 3(2a_1 + 48a_3 + 75a_4 + 3a_6) w_{1,1} w_{3,1} w_{5,1}.
\end{aligned}$$

The subscripted a's have the following significance:

$$\begin{aligned}
a_1 &= \alpha^2 & a_9 &= \alpha^2 / (9\alpha^2 + 1)^2 \\
a_2 &= 1/\alpha^2 & a_{10} &= \alpha^2 / (16\alpha^2 + 1)^2 \\
a_3 &= \alpha^2 / (\alpha^2 + 1)^2 & a_{11} &= \alpha^2 / (25\alpha^2 + 1)^2 \\
a_4 &= \alpha^2 / (\alpha^2 + 4)^2 & a_{12} &= \alpha^2 / (4\alpha^2 + 9)^2 \\
a_5 &= \alpha^2 / (\alpha^2 + 9)^2 & a_{13} &= \alpha^2 / (9\alpha^2 + 4)^2 \\
a_6 &= \alpha^2 / (\alpha^2 + 16)^2 & a_{14} &= \alpha^2 / (\alpha^2 + 36)^2 \\
a_7 &= \alpha^2 / (\alpha^2 + 25)^2 & a_{15} &= \alpha^2 / (\alpha^2 + 49)^2 \\
a_8 &= \alpha^2 (4\alpha^2 + 1)^2
\end{aligned}$$

where  $\alpha = a/b$ .





M3888-3401,FMS,DEBUG,1,1,500,0

BRILLANTES-LIVADAS

## SOLUTION OF THE DEFLECTION EQUATION

$$W = W_{1,1} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} + W_{3,1} \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b} \\ + W_{5,1} \sin \frac{5\pi x}{a} \sin \frac{\pi y}{b} + W_{7,1} \sin \frac{7\pi x}{a} \sin \frac{\pi y}{b}$$

THIS METHOD IS SET UP FOR A CONTINUOUS  
CHANGE IN AXIAL LOAD FROM ZERO TO A VALUE WITHIN  
THE VICINITY OF THE CRITICAL BUCKLING LOAD. ZERO  
LATERAL LOAD IS EXCLUDED.

## MAIN PROGRAM

```

*      XEQ
*
*      LABEL
*
*      LIST8
*
*      FORTRAN
*
      READ 1, IND
1  FORMAT (I5)
      PRINT 2
2  FORMAT (82H1  PNORM  AXIAL  W(1,1)/H  W(3,1)/H  W(5,1)/H  W(7,
11)/H  STRAIN  ERROR  )
      DO 44  JOB = 1,IND
      READ 3, U, ALPHA, PNORM, LOOP, LAP
3  FORMAT (3F8.0,2I8)
      DIMENSION B1(4), B2(4), B3(4)
      CALL CONST (U, ALPHA, PNORM, A1, A2, A3, A4, A5, A6, A7, A8, A9,
1      A10, A11, A12, A13, A14, A15, A16, A17, B1, B2, B3)
      INITIAL ESTIMATES OF W(1)'S

```



DIMENSION W(4)

W(1) = 0.400

W(2) = 0.300

W(3) = 0.200

W(4) = 0.100

THE FOLLOWING IS THE ROUTINE FOR  
CALCULATING THE DEFLECTION COEFFICIENTS FOR  
CHOSEN VALUES OF AXIAL LOAD AND NORMAL LOAD

DIMENSION A(4,4), C(4), X(4), Z(3), S(3,4)

AX = 0.

MG = 1

DO 10 LL = 1,3

4 CALL SESAME

CALL CROUT (A, C, X, M)

GO TO (5, 42), M

5 ERROR = 0.

DO 6 J = 1,4

6 ERROR = ERROR + ABSF(X(J))

IF (ERROR - 0.0001) 9, 9, 7

7 DO 8 I = 1,4

8 W(I) = W(I) + X(I)

GO TO 4

9 CALL CPRNT (PNORM, AX, W, ERROR, A1)

CALL STORE (AX, W, Z, S, LL)

AX = AX + 0.5

10 CONTINUE

DEL = 0.5

KO = 1





22 GO TO (23, 25, 27), KO

23 DEL = 0.05

DO 24 I = 1,4

W(I) = S(2,I)

24 S(3,I) = W(I)

AX = Z(2)

Z(3) = AX

AX = AX + DEL

KO = 2

KK = 0

CALL EXCH (Z, S)

GO TO 13

25 DEL = 0.01

DO 26 I = 1,4

W(I) = S(2,I)

26 S(3,I) = W(I)

AX = Z(2)

Z(3) = AX

AX = AX + DEL

KO = 3

KK = 0

CALL EXCH (Z, S)

GO TO 13

C THE FOLLOWING IS THE ROUTINE WHEN IN THE  
C VICINITY OF THE BUCKLING LOAD. W(1) IS MADE  
C THE INDEPENDENT VARIABLE IN PLACE OF THE AXIAL  
C LOAD.

27 DO 28 I = 1,4





```

31 ERROR = 0.
   DO 32 J = 1,4
32 ERROR = ERROR + ARSF(X(J))
   IF (ERROR - 0.0001) 36, 36, 33
33 KK = KK + 1
   IF (KK - LOOP) 34, 34, 40
34 DO 35 I = 2,4
35 W(I) = W(I) + X(I)
   AX = AX + X(1)
   GO TO 30
36 CALL CPRNT (PNORM, AX, W, ERROR, A1)
   IF (AX - Z(2)) 44, 44, 37
37 Z(3) = AX
   DO 38 I = 1,4
38 S(3,I) = W(I)
   DL1 = Z(3) - Z(2)
39 CONTINUE
   GO TO 44
40 PRINT 41
41 FORMAT (40H INFINITE LOOP OR NEEDS MORE ITERATION)
   CALL CPRNT (PNORM, AX, W, ERROR, A1)
   GO TO 44
42 PRINT 43
43 FORMAT (19H CROUT WENT CRAZY)
44 CONTINUE
   CALL EXIT
   COMMON W, AX, B1, B2, B3, A1, A2, A3, A4, A5, A6, A7, A8, A9, A10,
1      A11, A12, A13, A14, A15, A16, A17, MG, A, C

```



END

# SUBROUTINES

SUBROUTINE FOR SOLVING THE LINEARIZED SYSTEM  
OF EQUATIONS BY THE CROUT REDOX METHOD

\* LABEL

\* LIST8

\* FORTRAN

SUBROUTINE CROUT (A, C, X, M)

DIMENSION A(4,4), C(4), AA(4,4), CC(4), X(4)

DO 100 I = 1,4

100 AA(I,1) = A(I,1)

DO 101 J = 2,4

101 AA(1,J) = A(1,J)/A(1,1)

DO 102 I = 2,4

DO 102 J = 2,4

102 AA(I,J) = 0.

J = 2

103 II = J

DO 105 I = II,4

LIM1 = J - 1

DO 104 K = 1,LIM1

104 AA(I,J) = AA(I,J) + (AA(I,K) \* AA(K,J))

105 AA(I,J) = A(I,J) - AA(I,J)

IF (AA(J,J)) 106, 116, 106

106 IF (4 - J) 110, 110, 107

107 I = J



THEORY

Let  $f(x)$  be a function defined on the interval  $[a, b]$ . Then the definite integral of  $f(x)$  from  $a$  to  $b$  is denoted by  $\int_a^b f(x) dx$ .

where

$f(x)$  is the function to be integrated.

$a$  and  $b$  are the limits of integration.

The definite integral is a number, and it represents the area under the curve  $y = f(x)$  from  $x = a$  to  $x = b$ .

It is important to note that the definite integral is not a function of  $x$ , but a function of the limits  $a$  and  $b$ .

For example, if  $f(x) = x^2$ , then

$$\int_a^b x^2 dx = \frac{1}{3}x^3 \Big|_a^b = \frac{1}{3}(b^3 - a^3).$$

which is a number, and it represents the area under the curve  $y = x^2$  from  $x = a$  to  $x = b$ .

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The definite integral is a number, and it represents the area under the curve  $y = f(x)$  from  $x = a$  to  $x = b$ .

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It is also important to note that the definite integral is not a function of  $x$ , but a function of the limits  $a$  and  $b$ .

where

```

J = J + 1
JJ = J
DO 109 J = JJ,4
LIM2 = I - 1
DO 108 K = 1,LIM2
108 AA(I,J) = AA(I,J) + (AA(I,K) * AA(K,J))
AA(I,J) = A(I,J) - AA(I,J)
109 AA(I,J) = AA(I,J)/AA(I,I)
J = I + 1
GO TO 103
110 DO 111 I = 1,4
X(I) = 0.
111 CC(I) = 0.
CC(1) = C(1)/AA(1,1)
DO 113 I = 2,4
LIM3 = I - 1
DO 112 K = 1,LIM3
112 CC(I) = CC(I) + (AA(I,K) * CC(K))
CC(I) = C(I) - CC(I)
113 CC(I) = CC(I)/AA(I,I)
X(4) = CC(4)
DO 115 I = 1,3
II = 4 - I
LIM4 = II + 1
DO 114 K = LIM4,4
114 X(II) = X(II) + (AA(II,K) * X(K))
115 X(II) = CC(II) - X(II)
M = 1

```





```

      GO TO 118
116 PRINT 117
117 FORMAT (28H SINGULARITY - NO SOLUTION)
      M = 2
118 RETURN
      END

```

```

C          SUBROUTINE FOR CALCULATING THE NON-DIMENSIONAL
C          STRAIN AND THEN PRINTING ALL THE NEEDED RESULTS

```

```

* LABEL
* LIST8
* FORTRAN
SUBROUTINE CPRNT (PNORM, AX, W, ERROR, A1)
DIMENSION W(4)
SUM = 0.
DO 200 IJ = 1,4
  XY = 2 * IJ - 1
200 SUM = SUM + (XY * W(IJ))**2
  STRAIN = AX + 3.1416**2 * SUM/(8. * A1)
  PRINT 201, PNORM, AX, (W(I), I=1,4), STRAIN, ERROR
201 FORMAT (F8.2, F11.5, 4F10.3, F11.5, F12.5)
  RETURN
  END

```

```

C          SUBROUTINE FOR STORAGE OF THREE SETS OF VALUES
C          OF AXIAL LOAD AND THE FOUR DEFLECTION COEFFICIENTS.

```

```

* LABEL
* LIST8
* FORTRAN
SUBROUTINE STORF (AX, W, Z, S, LL)

```

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```
DIMENSION W(4), Z(3), S(3,4)
```

```
Z(LL) = AX
```

```
DO 203 I = 1,4
```

```
203 S(LL,I) = W(I)
```

```
RETURN
```

```
END
```

THIS SUBROUTINE IS JUST A SHIFTING PROCEDURE  
OF VALUES FROM ONE STORAGE LOCATION TO ANOTHER.

```
* LABEL
```

```
* LIST8
```

```
* FORTRAN
```

```
SUBROUTINE EXCH (Z, S)
```

```
DIMENSION Z(3), S(3,4)
```

```
DO 204 I = 1,4
```

```
DO 204 J = 1,2
```

```
JA = J + 1
```

```
204 S(J,I) = S(JA,I)
```

```
DO 205 IJ = 1,2
```

```
JB = IJ + 1
```

```
205 Z(IJ) = Z(JB)
```

```
DO 206 JI = 1,4
```

```
206 S(3,JI) = 0.
```

```
Z(3) = 0.
```

```
RETURN
```

```
END
```

SUBROUTINE FOR COMPUTING ALL THE NECESSARY  
CONSTANTS IN THE MAIN PROGRAM.

```
* LABEL
```





LIST8

FORTRAN

```

SUBROUTINE CONST (U, ALPHA, PNORM, A1, A2, A3, A4, A5, A6, A7, A8,
1A9, A10, A11, A12, A13, A14, A15, A16, A17, B1, B2, B3)
  DIMENSION B1(4), B2(4), B3(4), P(9)
  P(1) = ALPHA**2
  P(2) = 1./P(1)
  DO 300 I = 1,7
    Y1 = I**2
    I1 = I + 2
300 P(I1) = P(1)/(P(1) + Y1)**2
    A1 = P(1)
    A2 = P(1) + P(2)
    A3 = P(1) + 81. * P(2)
    A4 = P(1) + 625. * P(2)
    A5 = P(1) + 24(1. * P(2)
    A6 = 3. * P(1) + 64. * P(3)
    A7 = 3. * P(1) + 576. * P(3)
    A8 = 3. * P(1) + 9. * P(4)
    A9 = 3. * P(1) + 25. * P(4)
    A10 = 3. * P(1) + 64. * P(5)
    A11 = 4. * P(1) + 9. * P(2) + 16. * P(3) + P(4)
    A12 = 4. * P(1) + 225. * P(2) + 256. * P(3) + P(6)
    A13 = 4. * P(1) + 1225. * P(2) + 1296. * P(3) + P(8)
    A14 = 4. * P(1) + 25. * P(2) + 81. * P(4) + 16. * P(5)
    A15 = 4. * P(1) + 441. * P(2) + 625. * P(4) + 16. * P(7)
    A16 = 4. * P(1) + 49. * P(2) + 256. * P(5) + 81. * P(6)
    A17 = 6. * P(1) + 144. * P(3) + 225. * P(4) + 9. * P(6)

```





```

DO 301 I = 1,4
  I2 = 2 * I + 1
  Y2 = 2 * I - 1
  B1(I) = 256. * P(1) * PNORM/(Y2 * 3.1416**6)
  B2(I) = 4./(3. * (1. - U**2) * P(I2))
301 B3(I) = -(16. * Y2**2)/(3.1416**2)
  RETURN
END

      SUBROUTINE FOR COMPUTING THE COEFFICIENTS
      FOR THE USE OF THE CRUT REDOX METHOD

*
* LABEL
*
* LIST8
*
* FORTRAN

SUBROUTINE SESAME
COMMON W, AX, B1, B2, B3, A1, A2, A3, A4, A5, A6, A7, A8, A9, A10,
1 A11, A12, A13, A14, A15, A16, A17, MG, A, C
DIMENSION W(4), B1(4), B2(4), B3(4), A(4,4), C(4)
GO TO (401, 400), MG

400 A(1,1)=B3(1)*W(1)
  A(2,1)=B3(2)*W(2)
  A(3,1)=B3(3)*W(3)
  A(4,1)=B3(4)*W(4)
  GO TO 402

401 A(1,1)=B2(1)+B3(1)*AX+3.*A2*W(1)**2-4.*A1*W(1)*W(2)+A11*W(2)**2
1 +A14*W(3)**2+A16*W(4)**2-2.*A8*W(2)*W(3)-2.*A10*W(3)*W(4)
  A(2,1)=-3.*A1*W(1)**2+2.*A11*W(1)*W(2)-2.*A8*W(1)*W(3)
1 +2.*A6*W(2)*W(3)-2.*A7*W(2)*W(4)+A17*W(3)*W(4)
  A(3,1)=-2.*A8*W(1)*W(2)+2.*A14*W(1)*W(3)-7.*A10*W(1)*W(4)

```

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1. The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation  $f(x) = \int_0^x f(t) dt$ . It is shown that  $f(x)$  is a constant function, and its value is determined by the initial condition  $f(0) = 1$ .

2. In the second part, we consider the problem of finding the maximum value of the function  $f(x)$  on the interval  $[0, 1]$ . It is shown that the maximum value is attained at  $x = 0$  and is equal to 1.

3. The third part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation  $f(x) = \int_0^x f(t) dt$ . It is shown that  $f(x)$  is a constant function, and its value is determined by the initial condition  $f(0) = 1$ .

4. In the fourth part, we consider the problem of finding the maximum value of the function  $f(x)$  on the interval  $[0, 1]$ . It is shown that the maximum value is attained at  $x = 0$  and is equal to 1.

5. The fifth part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation  $f(x) = \int_0^x f(t) dt$ . It is shown that  $f(x)$  is a constant function, and its value is determined by the initial condition  $f(0) = 1$ .

6. In the sixth part, we consider the problem of finding the maximum value of the function  $f(x)$  on the interval  $[0, 1]$ . It is shown that the maximum value is attained at  $x = 0$  and is equal to 1.

$$1 \quad +A6*W(2)**2+A17*W(2)*W(4)$$

$$A(4,1)=-2.*A10*W(1)*W(3)+2.*A16*W(1)*W(4)-A9*W(2)**2+A17*W(2)*W(3)$$

$$402 \quad A(1,2)=-3.*A1*W(1)**2+2.*A11*W(1)*W(2)+2.*A6*W(2)*W(3)$$

$$1 \quad -2.*A9*W(2)*W(4)-2.*A8*W(1)*W(3)+A17*W(3)*W(4)$$

$$A(2,2)=B2(2)+B3(2)*AX+A11*W(1)**2+2.*A3*W(2)**2+A12*W(3)**2$$

$$1 \quad +A15*W(4)**2+2.*A6*W(1)*W(3)-2.*A9*W(1)*W(4)$$

$$A(3,2)=-A8*W(1)**2+2.*A6*W(1)*W(2)+2.*A12*W(2)*W(3)$$

$$1 \quad +A17*W(1)*W(4)+2.*A7*W(3)*W(4)$$

$$A(4,2)=-2.*A9*W(1)*W(2)+2.*A15*W(2)*W(4)+A7*W(3)**2+A17*W(1)*W(3)$$

$$A(1,3)=A6*W(2)**2+2.*A14*W(1)*W(3)-2.*A8*W(1)*W(2)$$

$$1 \quad -2.*A10*W(1)*W(4)+A17*W(2)*W(4)$$

$$A(2,3)=-A9*W(1)**2+2.*A12*W(2)*W(3)+2.*A7*W(3)*W(4)$$

$$1 \quad +2.*A6*W(1)*W(2)+A17*W(1)*W(4)$$

$$A(3,3)=B2(3)+B3(3)*AX+A14*W(1)**2+A12*W(2)**2+3.*A4*W(3)**2$$

$$1 \quad +A13*W(4)**2+2.*A7*W(2)*W(4)$$

$$A(4,3)=-A10*W(1)**2+2.*A7*W(2)*W(3)+2.*A13*W(3)*W(4)+A17*W(1)*W(2)$$

$$A(1,4)=-A9*W(2)**2+2.*A16*W(1)*W(4)-2.*A10*W(1)*W(3)+A17*W(2)*W(3)$$

$$A(2,4)=A7*W(3)**2+2.*A15*W(2)*W(4)-2.*A9*W(1)*W(2)+A17*W(1)*W(3)$$

$$A(3,4)=-A10*W(1)**2+2.*A13*W(3)*W(4)+A17*W(1)*W(2)+2.*A7*W(2)*W(3)$$

$$A(4,4)=B2(4)+B3(4)*AX+A16*W(1)**2+A15*W(2)**2+A12*W(3)**2$$

$$1 \quad +3.*A5*W(4)**2$$

$$C(1)=B1(1)-B2(1)*W(1)-B3(1)*W(1)*AX-A2*W(1)**2+3.*A1*W(1)**2*W(2)$$

$$1 \quad -A11*W(1)*W(2)**2-A6*W(2)**2*W(3)+A9*W(2)**2*W(4)$$

$$2 \quad -A14*W(1)*W(3)**2-A16*W(1)*W(4)**2+2.*A8*W(1)*W(2)*W(3)$$

$$3 \quad +2.*A10*W(1)*W(3)*W(4)-A17*W(2)*W(3)*W(4)$$

$$C(2)=B1(2)-B2(2)*W(2)-B3(2)*AX*W(2)+A1*W(1)**2-A11*W(1)**2*W(2)$$

$$1 \quad +A8*W(1)**2*W(2)-A3*W(2)**2-A12*W(2)*W(3)**2-A7*W(3)**2*W(4)$$

$$2 \quad -A15*W(2)*W(4)**2-2.*A6*W(1)*W(2)*W(3)+2.*A9*W(1)*W(2)*W(4)$$





```

3      -A17*W(1)*W(3)*W(4)

C(3)=B1(3)-B2(3)*W(3)-B3(3)*AX*W(3)+A8*W(1)**2*W(2)

1      -A14*W(1)**2*W(3)+A10*W(1)**2*W(4)-A6*W(1)*W(2)**2
2      -A12*W(2)**2*W(3)-A4*W(3)**3-A12*W(2)*W(4)**2
3      -A17*W(1)*W(2)*W(4)-2.*A7*W(2)*W(3)*W(4)

C(4)=P1(4)-B2(4)*W(4)-B3(4)*AX*W(4)+A10*W(1)**2*W(3)

1      -A16*W(1)**2*W(4)+A9*W(1)*W(2)**2-A15*W(2)**2*W(4)
2      -A7*W(2)*W(3)**2-A13*W(3)**2*W(4)-A5*W(4)**3
3      -A17*W(1)*W(2)*W(3)

RETURN

END

```





M3888-3401,FMS,DFBUG,1,1,500,0

# SOLUTION OF THE DEFLECTION EQUATION

$$W = W_{1,1} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} + W_{1,3} \sin \frac{\pi x}{a} \sin \frac{3\pi y}{b} \\ + W_{3,1} \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b} + W_{3,3} \sin \frac{3\pi x}{a} \sin \frac{3\pi y}{b} \\ + W_{1,5} \sin \frac{\pi x}{a} \sin \frac{5\pi y}{b} + W_{5,1} \sin \frac{5\pi x}{a} \sin \frac{\pi y}{b}$$

THIS METHOD IS SET UP FOR A CONTINUOUS  
CHANGE IN AXIAL LOAD FROM ZERO TO A VALUE WITHIN  
THE VICINITY OF THE CRITICAL BUCKLING LOAD. ZERO  
LATERAL LOAD IS EXCLUDED.

## MAIN PROGRAM

```
XEQ
*
* LABEL
*
* LIST8
*
* FORTRAN
*
READ 1, IND
1 FORMAT (I5)
PRINT 2
2 FORMAT (114H1 PNORM AXIAL W(1,1)/H W(1,3)/H W(3,1)/
1H W(3,3)/H W(1,5)/H W(5,1)/H STRAIN ERROR )
DO 44 JOB = 1,IND
READ 3, U, ALPHA, PNORM, LOOP, LAP
3 FORMAT (3F8.0,2I8)
DIMENSION W(6), R(54), B1(4), B2(5), B3(3), A(6,6), C(6), X(6)
CALL CONST (U, ALPHA, PNORM, R, B1, B2, B3)
INITIAL ESTIMATES OF W(I)'S
```



W(1) = 0.100

W(2) = 0.010

W(3) = 0.010

W(4) = 0.001

W(5) = 0.001

W(6) = 0.001

THE FOLLOWING IS THE ROUTINE FOR  
CALCULATING THE DEFLECTION COEFFICIENTS FOR  
CHOSEN VALUES OF AXIAL LOAD AND NORMAL LOAD

DIMENSION Z(3), S(3,6)

AX = 0.

MG = 1

RONE = R(1)

DO 10 LL = 1,3

4 CALL SFSAME

CALL CROUT (A, C, X, M)

GO TO (5, 42), M

5 ERROR = 0.

DO 6 J = 1,6

6 ERROR = ERROR + ABSF(X(J))

IF (ERROR - 0.000001) 9, 9, 7

7 DO 8 I = 1,6

8 W(I) = W(I) + X(I)

GO TO 4

9 CALL CPRNT (PNORM, AX, W, ERROR, RONE)

CALL STORE (AX, W, Z, S, LL)

AX = AX + 0.5

10 CONTINUE





```

DEL = 0.5
KO = 1
KK = 0

11 DO 12 I = 1,6
    F1 = (S(2,I) - S(1,I))/(Z(2) - Z(1))
    F2 = (S(3,I) - S(2,I))/(Z(3) - Z(2))
    F3 = (Z(3) - Z(2))/2.
    F4 = (F2 - F1)/F3
12 W(I) = W(I) + F2 * DEL + 0.5 * F4 * DEL**2
    CALL EXCH (Z, S)
13 CALL SESAME
    CALL CROUT (A, C, X, M)
    GO TO (14, 42), M
14 ERROR = 0.
    DO 15 J = 1,6
15 ERROR = ERROR + ABSF(X(J))
    IF (ERROR - 0.000001) 19, 19, 16
16 KK = KK + 1
    IF (KK - LOOP) 17, 17, 40
17 DO 18 I = 1,6
18 W(I) = W(I) + X(I)
    GO TO 13
19 IF (W(1) - S(2,1)) 22, 22, 20
20 CALL CPRNT (PNORM, AX, W, ERROR, RONE)
    Z(3) = AX
    DO 21 I = 1,6
21 S(3,I) = W(I)
    AX = AX + DEL

```





```

KK = 0
GO TO 11
22 GO TO (23, 25, 27), KO
23 DEL = 0.05
DO 24 I = 1,6
W(I) = S(2,I)
24 S(3,I) = W(I)
AX = Z(2)
Z(3) = AX
AX = AX + DEL
KO = 2
KK = 0
CALL EXCH (Z, S)
GO TO 13
25 DEL = 0.01
DO 26 I = 1,6
W(I) = S(2,I)
26 S(3,I) = W(I)
AX = Z(2)
Z(3) = AX
AX = AX + DEL
KO = 3
KK = 0
CALL EXCH (Z, S)
GO TO 13

```

C THE FOLLOWING IS THE ROUTINE WHEN IN THE  
 C VICINITY OF THE BUCKLING LOAD. W(1) IS MADE  
 C THE INDEPENDENT VARIABLE IN PLACE OF THE AXIAL



LOAD.

27 DO 28 I = 1,6

S(3,I) = S(2,I)

S(2,I) = S(1,I)

28 W(I) = S(3,I)

Z(3) = Z(2)

Z(2) = Z(1)

AX = Z(3)

THE FOLLOWING VALUES OF DL1 AND DL2 ARE ARBITRARY.

THE FIRST VALUE OF DL1 IS SO CHOSEN SUCH THAT WHEN

W(1) IS DECREASED BY DL2, THE CONSEQUENT ITERATED

SOLUTION WILL NOT GO BACK ON THE PREVIOUSLY COMPUTED

PATH. (NOTE - THE W(1) OR THE FIRST HARMONIC STARTS

TO DECREASE IN VALUE BEFORE THE CRITICAL BUCKLING

LOAD IS REACHED)

DL1 = 0.20

DL2 = 0.010

.MG = 2

THIS LAST DO LOOP IS HOPED TO GO BEYOND THE CRI-

TICAL RANGE. START WITH LAP = 30.

DO 39 LAST = 1,LAP

KK = 0

AX = AX + DL1

W(1) = W(1) - DL2

DO 29 J = 2,6

29 W(I) = W(I) + (S(3,J) - S(2,J))

CALL EXCH (Z, S)

30 CALL SESAME





```

      CALL CROUT (A, C, X, M)
      GO TO (31, 42), M
31  ERROR = 0.
      DO 32  J = 1,6
32  ERROR = ERROR + ABSF(X(J))
      IF (ERROR - 0.000001) 36, 36, 33
33  KK = KK + 1
      IF (KK - LOOP) 34, 34, 40
34  DO 35  I = 2,6
35  W(I) = W(I) + X(I)
      AX = AX + X(1)
      GO TO 30
36  CALL CPRNT (PNORM, AX, W, ERROR, RONE)
      IF (AX - Z(2)) 44, 44, 37
37  Z(3) = AX
      DO 38  I = 1,6
38  S(3,I) = W(I)
      DL1 = Z(3) - Z(2)
39  CONTINUE
      GO TO 44
40  PRINT 41
41  FORMAT (40H  INFINITE LOOP OR NEEDS MORE ITERATION)
      CALL CPRNT (PNORM, AX, W, ERROR, RONE)
      GO TO 44
42  PRINT 43
43  FORMAT (19H  CROUT WENT CRAZY)
44  CONTINUE
      CALL EXIT

```





```
COMMON W, AX, B1, B2, B3, R, MG, A, C
```

```
END
```

# SUBROUTINES

SUBROUTINE FOR SOLVING THE LINEARIZED SYSTEM  
OF EQUATIONS BY THE CROUT REDOX METHOD

```
* LABEL
```

```
* LIST8
```

```
* FORTRAN
```

```
SUBROUTINE CROUT (A, C, X, M)
```

```
DIMENSION A(6,6), C(6), AA(6,6), CC(6), X(6)
```

```
DO 100 I = 1,6
```

```
100 AA(I,1) = A(I,1)
```

```
DO 101 J = 2,6
```

```
101 AA(1,J) = A(1,J)/A(1,1)
```

```
DO 102 I = 2,6
```

```
DO 102 J = 2,6
```

```
102 AA(I,J) = 0.
```

```
J = 2
```

```
103 II = J
```

```
DO 105 I = II,6
```

```
LIM1 = J - 1
```

```
DO 104 K = 1,LIM1
```

```
104 AA(I,J) = AA(I,J) + (AA(I,K) * AA(K,J))
```

```
105 AA(I,J) = A(I,J) - AA(I,J)
```

```
IF (AA(J,J)) 106, 116, 106
```

```
106 IF (6 - J) 110, 110, 107
```



```

107 I = J
      J = J + 1
      JJ = J
      DO 109 J = JJ,6
      LIM2 = I - 1
      DO 108 K = 1,LIM2
108 AA(I,J) = AA(I,J) + (AA(I,K) * AA(K,J))
      AA(I,J) = A(I,J) - AA(I,J)
109 AA(I,J) = AA(I,J)/AA(I,I)
      J = I + 1
      GO TO 103
110 DO 111 I = 1,6
      X(I) = 0.
111 CC(I) = 0.
      CC(1) = C(1)/AA(1,1)
      DO 113 I = 2,6
      LIM3 = I - 1
      DO 112 K = 1,LIM3
112 CC(I) = CC(I) + (AA(I,K) * CC(K))
      CC(I) = C(I) - CC(I)
113 CC(I) = CC(I)/AA(I,I)
      X(6) = CC(6)
      DO 115 I = 1,5
      II = 6 - I
      LIM4 = II + 1
      DO 114 K = LIM4,6
114 X(II) = X(II) + (AA(II,K) * X(K))
115 X(II) = CC(II) - X(II)

```





M = 1

GO TO 118

116 PRINT 117

117 FORMAT (28H SINGULARITY - NO SOLUTION)

M = 2

118 RETURN

END

SUBROUTINE FOR STORAGE OF THREE SETS OF VALUES  
OF AXIAL LOAD AND THE FOUR DEFLECTION COEFFICIENTS.

\* LABEL

\* LIST8

\* FORTRAN

SUBROUTINE STORE (AX, W, Z, S, LL)

DIMENSION W(6), Z(3), S(3,6)

Z(LL) = AX

DO 203 I = 1,6

203 S(LL,I) = W(I)

RETURN

END

THIS SUBROUTINE IS JUST A SHIFTING PROCEDURE  
OF VALUES FROM ONE STORAGE LOCATION TO ANOTHER.

\* LABEL

\* LIST8

\* FORTRAN

SUBROUTINE EXCH (Z, S)

DIMENSION Z(3), S(3,6)

DO 204 I = 1,6

DO 204 J = 1,2





```

      JA = J + 1
204 S(J,I) = S(JA,I)
      DO 205 IJ = 1,2
      JB = IJ + 1
205 Z(IJ) = Z(JB)
      DO 206 JI = 1,6
206 S(3,JI) = 0.
      Z(3) = 0.
      RETURN
      END

C      SUBROUTINE FOR PRINTING
*
* LABEL
*
* LIST8
*
* FORTRAN
      SUBROUTINE CPRNT (PNORM, AX, W, ERROR, RONE)
      DIMENSION W(6)
      SUM=W(1)**2+W(2)**2+W(5)**2+9.*(W(3)**2+W(4)**2)+25.*W(6)**2
      STRAIN = AX + 3.1416**2 * SUM/(8. * RONE)
      PRINT 200, PNORM, AX, (W(I), I=1,6), STRAIN, ERROR
200 FORMAT (F8.2, F11.5, 6F12.6, F11.5, F12.7)
      RETURN
      END

C      SUBROUTINE FOR COMPUTING ALL THE NECESSARY
C      CONSTANTS IN THE MAIN PROGRAM
*
* LABEL
*
* LIST8
*
* FORTRAN
      SUBROUTINE CONST (U, ALPHA, PNORM, R, B1, B2, B3)

```



```
DIMENSION P(13), R(54), B1(4), B2(5), B3(3)
```

```
P(1) = ALPHA**2
```

```
P(2) = 1./P(1)
```

```
DO 300 I = 3,7
```

```
Y1 = (I - 2)**2
```

```
300 P(I) = P(1)/(P(1) + Y1)**2
```

```
DO 301 J = 8,11
```

```
Y2 = (J - 6)**2
```

```
301 P(J) = P(1)/(1. + Y2 * P(1))**2
```

```
P(12) = P(1)/(4. * P(1) + 9. )**2
```

```
P(13) = P(1)/(4. + 9. * P(1))**2
```

```
R(1) = P(1)
```

```
R(2) = P(2)
```

```
R(3) = P(3)
```

```
R(4) = P(4)
```

```
R(5) = P(8)
```

```
R(6) = P(1) + P(2)
```

```
R(7) = P(1) + 81.*P(2)
```

```
R(8) = P(1) + 625.*P(2)
```

```
R(9) = 81.*P(1) + P(2)
```

```
R(10) = 625.*P(1) + P(2)
```

```
R(11) = P(1) + P(8)
```

```
R(12) = P(1) + 81.*P(8)
```

```
R(13) = P(2) + P(4)
```

```
R(14) = P(2) + 81.*P(4)
```

```
R(15) = P(1) + 3.*P(4)
```

```
R(16) = P(2) + 3.*P(8)
```

```
R(17) = 3.*P(1) + 64.*P(3)
```





$$R(18) = 3.*P(2) + 64.*P(3)$$

$$R(19) = 64.*P(2) + 274.*P(4)$$

$$R(20) = 64.*P(3) + 274.*P(8)$$

$$R(21) = P(4) + P(8)$$

$$R(22) = P(3) + P(5)$$

$$R(23) = P(3) + P(9)$$

$$R(24) = 9.*P(1) + 64.*P(3) + 25.*P(8)$$

$$R(25) = 9.*P(2) + 64.*P(3) + 25.*P(4)$$

$$R(26) = 25.*P(1) + 441.*P(8) + 256.*P(9)$$

$$R(27) = 25.*P(2) + 441.*P(4) + 256.*P(5)$$

$$R(28) = 272.*P(3) + 625.*P(4) + 625.*P(8)$$

$$R(29) = 9.*P(1) + 64.*P(3) + 1225.*P(4)$$

$$R(30) = 9.*P(2) + 64.*P(3) + 1225.*P(8)$$

$$R(31) = 25.*P(1) + 64.*P(3) + 9.*P(10)$$

$$R(32) = 25.*P(2) + 64.*P(3) + 9.*P(6)$$

$$R(33) = 64.*P(3) + 441.*P(4) + 169.*P(13)$$

$$R(34) = 64.*P(3) + 441.*P(8) + 169.*P(12)$$

$$R(35) = 25.*P(1) + 576.*P(3) + 8281.*P(13)$$

$$R(36) = 25.*P(2) + 576.*P(3) + 8281.*P(12)$$

$$R(37) = 1552.*P(3) + 28561.*P(12) + 28561.*P(13)$$

$$R(38) = 9.*P(1) + 4.*P(2) + 16.*P(3) + P(8)$$

$$R(39) = 4.*P(1) + 9.*P(2) + 16.*P(3) + P(4)$$

$$R(40) = 36.*P(1) + 36.*P(2) + 225.*P(4) + 225.*P(8)$$

$$R(41) = 25.*P(1) + 4.*P(2) + 81.*P(8) + 16.*P(9)$$

$$R(42) = 4.*P(1) + 25.*P(2) + 81.*P(4) + 16.*P(5)$$

$$R(43) = 36.*P(1) + P(2) + 144.*P(9) + 9.*P(13)$$

$$R(44) = P(1) + 36.*P(2) + 144.*P(5) + 9.*P(12)$$

$$R(45) = 2.*P(1) + 16.*P(3) + 49.*P(4) + 9.*P(8)$$





```

R(46) = 2.*P(2) + 16.*P(3) + 9.*P(4) + 49.*P(8)
R(47) = 16.*P(3) + 81.*P(4) + P(8) + 81.*P(10)
R(48) = 16.*P(3) + P(4) + 81.*P(6) + 81.*P(8)
R(49) = 4.*P(1) + 225.*P(2) + 256.*P(3) + P(6)
R(50) = 225.*P(1) + 4.*P(2) + 256.*P(3) + P(10)
R(51) = 256.*P(3) + 2401.*P(4) + 4096.*P(5) + 2401.*P(12)
R(52) = 256.*P(3) + 2401.*P(8) + 4096.*P(9) + 2401.*P(13)
R(53) = 2.*P(1) + 64.*P(3) + 256.*P(5) + 225.*P(8) + 49.*P(12)
R(54) = 2.*P(2) + 64.*P(3) + 225.*P(4) + 256.*P(9) + 49.*P(13)
DO 302 I = 1,3
Y3 = 2 * I - 1
B1(I) = 256. * P(1) * PNORM/(Y3 * 3.1416**6)
302 B3(I) = -(16. * Y3**2)/(3.1416**2)
B1(4) = B1(2)/3.
DO 303 J = 1,5
I1 = 2 * J + 1
303 B2(J) = 4./(3. * (1. - U**2) * P(I1))
RETURN
END

```

SUBROUTINE FOR COMPUTING THE COEFFICIENTS  
 FOR THE USE OF THE CROUT REDOX METHOD

\* LABEL

\* LIST8

\* FORTRAN

SUBROUTINE SESAME

COMMON W, AX, B1, B2, B3, R, MG, A, C

DIMENSION W(6), B1(4), B2(5), B3(3), R(54), A(6,6), C(6)

GO TO (401, 400), MG



400 A(1,1)=B3(1)\*W(1)

A(2,1)=B3(1)\*W(2)

A(3,1)=B3(2)\*W(3)

A(4,1)=B3(2)\*W(4)

A(5,1)=B3(1)\*W(5)

A(6,1)=B3(3)\*W(6)

GO TO 402

401 A(1,1)=B2(1)+B3(1)\*AX+3.\*R(6)\*W(1)\*\*2-6.\*R(2)\*W(1)\*W(2)

1 -6.\*R(1)\*W(1)\*W(3)+R(38)\*W(2)\*\*2+R(39)\*W(3)\*\*2

2 +32.\*R(3)\*W(2)\*W(3)+81.\*R(21)\*W(4)\*\*2-18.\*R(11)\*W(2)\*W(4)

3 -18.\*R(13)\*W(3)\*W(4)-6.\*R(16)\*W(2)\*W(5)-6.\*R(15)\*W(3)\*W(6)

4 +162.\*R(5)\*W(4)\*W(5)+162.\*R(4)\*W(4)\*W(6)+R(41)\*W(5)\*\*2

5 +R(42)\*W(6)\*\*2

A(2,1)=-3.\*R(2)\*W(1)\*\*2+2.\*R(38)\*W(1)\*W(2)+32.\*R(3)\*W(1)\*W(3)

1 -18.\*R(11)\*W(1)\*W(4)-R(25)\*W(2)\*\*2-2.\*R(24)\*W(2)\*W(3)

2 +R(40)\*W(3)\*W(4)-6.\*R(16)\*W(1)\*W(5)+2.\*R(18)\*W(2)\*W(5)

3 +R(20)\*W(3)\*W(5)+R(19)\*W(3)\*W(6)-144.\*R(23)\*W(4)\*W(5)

4 -9.\*R(45)\*W(4)\*W(6)-R(27)\*W(6)\*\*2

A(3,1)=-3.\*R(1)\*W(1)\*\*2+32.\*R(3)\*W(1)\*W(2)+2.\*R(39)\*W(1)\*W(3)

1 -18.\*R(13)\*W(1)\*W(4)-R(24)\*W(2)\*\*2-2.\*R(25)\*W(2)\*W(3)

2 +R(40)\*W(2)\*W(4)-6.\*R(15)\*W(1)\*W(6)+R(20)\*W(2)\*W(5)

3 +R(19)\*W(2)\*W(6)+2.\*R(17)\*W(3)\*W(6)-9.\*R(46)\*W(4)\*W(5)

4 -144.\*R(22)\*W(4)\*W(6)-R(26)\*W(5)\*\*2

A(4,1)=-18.\*R(11)\*W(1)\*W(2)-18.\*R(13)\*W(1)\*W(3)

1 +162.\*R(21)\*W(1)\*W(4)+R(40)\*W(2)\*W(3)+162.\*R(5)\*W(1)\*W(5)

2 +162.\*R(4)\*W(1)\*W(6)-144.\*R(23)\*W(2)\*W(5)

3 -9.\*R(45)\*W(2)\*W(6)-9.\*R(46)\*W(3)\*W(5)-144.\*R(22)\*W(3)\*W(6)

4 +18.\*R(14)\*W(4)\*W(5)+18.\*R(12)\*W(4)\*W(6)





$$5 \quad +729 \cdot R(21) \cdot W(5) \cdot W(6)$$

$$A(5,1) = -6 \cdot R(16) \cdot W(1) \cdot W(2) + 162 \cdot R(5) \cdot W(1) \cdot W(4) + R(18) \cdot W(2) \cdot W(3) \cdot W(4)$$

$$1 \quad +R(20) \cdot W(2) \cdot W(3) + 9 \cdot R(14) \cdot W(4) \cdot W(5) - 144 \cdot R(23) \cdot W(2) \cdot W(4)$$

$$2 \quad -9 \cdot R(46) \cdot W(3) \cdot W(4) + 2 \cdot R(41) \cdot W(1) \cdot W(5) - 2 \cdot R(26) \cdot W(3) \cdot W(5)$$

$$3 \quad +729 \cdot R(21) \cdot W(4) \cdot W(6)$$

$$A(6,1) = -6 \cdot R(15) \cdot W(1) \cdot W(3) + 162 \cdot R(4) \cdot W(1) \cdot W(4) + R(17) \cdot W(3) \cdot W(4) \cdot W(5)$$

$$1 \quad +R(19) \cdot W(2) \cdot W(3) + 9 \cdot R(12) \cdot W(4) \cdot W(5) - 9 \cdot R(45) \cdot W(2) \cdot W(4)$$

$$2 \quad -144 \cdot R(22) \cdot W(3) \cdot W(4) + 2 \cdot R(42) \cdot W(1) \cdot W(6) - 2 \cdot R(27) \cdot W(2) \cdot W(6)$$

$$3 \quad +729 \cdot R(21) \cdot W(4) \cdot W(5)$$

$$402 \quad A(1,2) = -3 \cdot R(2) \cdot W(1) \cdot W(2) \cdot W(3) + 2 \cdot R(38) \cdot W(1) \cdot W(2) + 32 \cdot R(3) \cdot W(1) \cdot W(3)$$

$$1 \quad -18 \cdot R(11) \cdot W(1) \cdot W(4) - 2 \cdot R(24) \cdot W(2) \cdot W(3) - R(25) \cdot W(3) \cdot W(4) \cdot W(5)$$

$$2 \quad +R(40) \cdot W(3) \cdot W(4) - 6 \cdot R(16) \cdot W(1) \cdot W(5) + 2 \cdot R(18) \cdot W(2) \cdot W(5)$$

$$3 \quad +R(20) \cdot W(3) \cdot W(5) + R(19) \cdot W(3) \cdot W(6) - 144 \cdot R(23) \cdot W(4) \cdot W(5)$$

$$4 \quad -9 \cdot R(45) \cdot W(4) \cdot W(6) - R(27) \cdot W(6) \cdot W(5)$$

$$A(2,2) = B2(4) + B3(1) \cdot AX + R(38) \cdot W(1) \cdot W(2) \cdot W(3) - 2 \cdot R(24) \cdot W(1) \cdot W(3)$$

$$1 \quad +3 \cdot R(9) \cdot W(2) \cdot W(3) \cdot W(4) + R(28) \cdot W(3) \cdot W(4) \cdot W(5) - 486 \cdot R(1) \cdot W(2) \cdot W(4)$$

$$2 \quad +R(43) \cdot W(4) \cdot W(5) + 2 \cdot R(18) \cdot W(1) \cdot W(5) - 512 \cdot R(3) \cdot W(3) \cdot W(5)$$

$$3 \quad -2 \cdot R(29) \cdot W(3) \cdot W(6) + R(50) \cdot W(5) \cdot W(6) + R(51) \cdot W(6) \cdot W(5)$$

$$A(3,2) = 16 \cdot R(3) \cdot W(1) \cdot W(2) \cdot W(3) - 2 \cdot R(24) \cdot W(1) \cdot W(2) - 2 \cdot R(25) \cdot W(1) \cdot W(3)$$

$$1 \quad +R(40) \cdot W(1) \cdot W(4) + 2 \cdot R(28) \cdot W(2) \cdot W(3) + R(20) \cdot W(1) \cdot W(5)$$

$$2 \quad +R(19) \cdot W(1) \cdot W(6) - 512 \cdot R(3) \cdot W(2) \cdot W(5) - 2 \cdot R(29) \cdot W(2) \cdot W(6)$$

$$3 \quad -2 \cdot R(30) \cdot W(3) \cdot W(5) - 512 \cdot R(3) \cdot W(3) \cdot W(6) + 9 \cdot R(54) \cdot W(4) \cdot W(5)$$

$$4 \quad +9 \cdot R(53) \cdot W(4) \cdot W(6) + 656 \cdot R(3) \cdot W(5) \cdot W(6)$$

$$A(4,2) = -9 \cdot R(11) \cdot W(1) \cdot W(2) \cdot W(3) + R(40) \cdot W(1) \cdot W(3) - 43 \cdot R(1) \cdot W(2) \cdot W(3)$$

$$1 \quad +18 \cdot R(43) \cdot W(2) \cdot W(4) - 144 \cdot R(23) \cdot W(1) \cdot W(5) - 9 \cdot R(31) \cdot W(5) \cdot W(6)$$

$$2 \quad -9 \cdot R(45) \cdot W(1) \cdot W(6) + 9 \cdot R(54) \cdot W(3) \cdot W(5) + 9 \cdot R(53) \cdot W(3) \cdot W(6)$$

$$3 \quad -9 \cdot R(33) \cdot W(5) \cdot W(6)$$

$$A(5,2) = -3 \cdot R(16) \cdot W(1) \cdot W(2) \cdot W(3) + 2 \cdot R(18) \cdot W(1) \cdot W(2) + R(20) \cdot W(1) \cdot W(3)$$





$$\begin{aligned}
& 1 \quad -144 \cdot R(23) \cdot W(1) \cdot W(4) - 512 \cdot R(3) \cdot W(2) \cdot W(3) - R(30) \cdot W(3) \cdot W(3) \\
& 2 \quad + 9 \cdot R(54) \cdot W(3) \cdot W(4) + 2 \cdot R(50) \cdot W(2) \cdot W(5) + 656 \cdot R(3) \cdot W(3) \cdot W(6) \\
& 3 \quad - 18 \cdot R(31) \cdot W(4) \cdot W(5) - 9 \cdot R(33) \cdot W(4) \cdot W(6) - R(36) \cdot W(6) \cdot W(6)
\end{aligned}$$

$$A(6,2) = R(19) \cdot W(1) \cdot W(3) - 9 \cdot R(45) \cdot W(1) \cdot W(4) - 2 \cdot R(29) \cdot W(2) \cdot W(3)$$

$$\begin{aligned}
& 1 \quad -256 \cdot R(3) \cdot W(3) \cdot W(3) + 9 \cdot R(53) \cdot W(3) \cdot W(4) - 2 \cdot R(27) \cdot W(1) \cdot W(6) \\
& 2 \quad + 2 \cdot R(51) \cdot W(2) \cdot W(6) + 656 \cdot R(3) \cdot W(3) \cdot W(5) - 9 \cdot R(33) \cdot W(4) \cdot W(5) \\
& 3 \quad - 2 \cdot R(36) \cdot W(5) \cdot W(6)
\end{aligned}$$

$$A(1,3) = -3 \cdot R(1) \cdot W(1) \cdot W(1) + 2 \cdot R(39) \cdot W(1) \cdot W(3) + 32 \cdot R(3) \cdot W(1) \cdot W(2)$$

$$\begin{aligned}
& 1 \quad -18 \cdot R(13) \cdot W(1) \cdot W(4) - R(24) \cdot W(2) \cdot W(2) - 2 \cdot R(25) \cdot W(2) \cdot W(3) \\
& 2 \quad + R(40) \cdot W(2) \cdot W(4) - 6 \cdot R(15) \cdot W(1) \cdot W(6) + 2 \cdot R(17) \cdot W(3) \cdot W(6) \\
& 3 \quad + R(20) \cdot W(2) \cdot W(5) + R(19) \cdot W(2) \cdot W(6) - 9 \cdot R(46) \cdot W(4) \cdot W(5) \\
& 4 \quad - 144 \cdot R(22) \cdot W(4) \cdot W(6) - R(26) \cdot W(5) \cdot W(5)
\end{aligned}$$

$$A(2,3) = 16 \cdot R(3) \cdot W(1) \cdot W(1) - 2 \cdot R(25) \cdot W(1) \cdot W(3) - 2 \cdot R(24) \cdot W(1) \cdot W(2)$$

$$\begin{aligned}
& 1 \quad + R(40) \cdot W(1) \cdot W(4) + 2 \cdot R(28) \cdot W(2) \cdot W(3) + R(20) \cdot W(1) \cdot W(5) \\
& 2 \quad + R(19) \cdot W(1) \cdot W(6) - 2 \cdot R(30) \cdot W(3) \cdot W(5) - 512 \cdot R(3) \cdot W(3) \cdot W(6) \\
& 3 \quad - 512 \cdot R(3) \cdot W(2) \cdot W(5) - 2 \cdot R(29) \cdot W(2) \cdot W(6) + 9 \cdot R(54) \cdot W(4) \cdot W(5) \\
& 4 \quad + 9 \cdot R(53) \cdot W(4) \cdot W(6) + 656 \cdot R(3) \cdot W(5) \cdot W(6)
\end{aligned}$$

$$A(3,3) = B2(2) + B3(2) \cdot AX + R(39) \cdot W(1) \cdot W(1) - 2 \cdot R(25) \cdot W(1) \cdot W(2)$$

$$\begin{aligned}
& 1 \quad + R(28) \cdot W(2) \cdot W(2) + 3 \cdot R(7) \cdot W(3) \cdot W(3) - 486 \cdot R(2) \cdot W(3) \cdot W(4) \\
& 2 \quad + 9 \cdot R(44) \cdot W(4) \cdot W(4) + 2 \cdot R(17) \cdot W(1) \cdot W(6) - 2 \cdot R(30) \cdot W(2) \cdot W(5) \\
& 3 \quad - 512 \cdot R(3) \cdot W(2) \cdot W(6) + R(52) \cdot W(5) \cdot W(5) + R(49) \cdot W(6) \cdot W(6)
\end{aligned}$$

$$A(4,3) = -9 \cdot R(13) \cdot W(1) \cdot W(1) + R(40) \cdot W(1) \cdot W(2) - 243 \cdot R(2) \cdot W(3) \cdot W(3)$$

$$\begin{aligned}
& 1 \quad + 18 \cdot R(44) \cdot W(3) \cdot W(4) - 9 \cdot R(46) \cdot W(1) \cdot W(5) - 9 \cdot R(32) \cdot W(6) \cdot W(6) \\
& 2 \quad - 144 \cdot R(22) \cdot W(1) \cdot W(6) + 9 \cdot R(54) \cdot W(2) \cdot W(5) + 9 \cdot R(53) \cdot W(2) \cdot W(6) \\
& 3 \quad - 9 \cdot R(34) \cdot W(5) \cdot W(6)
\end{aligned}$$

$$A(5,3) = R(20) \cdot W(1) \cdot W(2) - 9 \cdot R(46) \cdot W(1) \cdot W(4) - 256 \cdot R(3) \cdot W(2) \cdot W(2)$$

$$\begin{aligned}
& 1 \quad - 2 \cdot R(30) \cdot W(2) \cdot W(3) + 9 \cdot R(54) \cdot W(2) \cdot W(4) - 2 \cdot R(26) \cdot W(1) \cdot W(5) \\
& 2 \quad + 2 \cdot R(52) \cdot W(3) \cdot W(5) + 656 \cdot R(3) \cdot W(2) \cdot W(6) - 9 \cdot R(34) \cdot W(4) \cdot W(6)
\end{aligned}$$



$$3 \quad -2.*R(35)*W(5)*W(6)$$

$$A(6,3)=-3.*R(15)*W(1)**2+2.*R(17)*W(1)*W(3)+R(19)*W(1)*W(2)$$

$$1 \quad -144.*R(22)*W(1)*W(4)-R(29)*W(2)**2-512.*R(3)*W(2)*W(3)$$

$$2 \quad +9.*R(53)*W(2)*W(4)+2.*R(49)*W(3)*W(6)+656.*R(3)*W(2)*W(5)$$

$$3 \quad -9.*R(34)*W(4)*W(5)-18.*R(32)*W(4)*W(6)-R(35)*W(5)**2$$

$$A(1,4)=162.*R(21)*W(1)*W(4)-18.*R(11)*W(1)*W(2)$$

$$1 \quad -18.*R(13)*W(1)*W(3)+R(40)*W(2)*W(3)+162.*R(5)*W(1)*W(5)$$

$$2 \quad +162.*R(4)*W(1)*W(6)-144.*R(23)*W(2)*W(5)$$

$$3 \quad -9.*R(45)*W(2)*W(6)-9.*R(46)*W(3)*W(5)-144.*R(22)*W(3)*W(6)$$

$$4 \quad +18.*R(14)*W(4)*W(5)+18.*R(12)*W(4)*W(6)$$

$$5 \quad -729.*R(21)*W(5)*W(6)$$

$$A(2,4)=-9.*R(11)*W(1)**2+R(40)*W(1)*W(3)-243.*R(1)*W(2)**2$$

$$1 \quad +2.*R(43)*W(2)*W(4)-144.*R(23)*W(1)*W(5)-9.*R(45)*W(1)*W(6)$$

$$2 \quad +9.*R(54)*W(3)*W(5)+9.*R(53)*W(3)*W(6)-9.*R(31)*W(5)**2$$

$$3 \quad -9.*R(33)*W(5)*W(6)$$

$$A(3,4)=-9.*R(13)*W(1)**2+R(40)*W(1)*W(2)-243.*R(2)*W(3)**2$$

$$1 \quad +18.*R(44)*W(3)*W(4)-9.*R(46)*W(1)*W(5)-9.*R(34)*W(5)*W(6)$$

$$2 \quad -144.*R(22)*W(1)*W(6)+9.*R(54)*W(2)*W(5)+9.*R(53)*W(2)*W(6)$$

$$3 \quad -9.*R(32)*W(6)**2$$

$$A(4,4)=81.*B2(1)+B3(2)*AX+81.*R(21)*W(1)**2+9.*R(43)*W(2)**2$$

$$1 \quad +243.*R(6)*W(4)**2+9.*R(44)*W(3)**2+18.*R(14)*W(1)*W(5)$$

$$2 \quad +18.*R(12)*W(1)*W(6)+81.*R(47)*W(5)**2+81.*R(48)*W(6)**2$$

$$3 \quad +2592.*R(3)*W(5)*W(6)$$

$$A(5,4)=81.*R(5)*W(1)**2+18.*R(14)*W(1)*W(4)+9.*R(54)*W(2)*W(3)$$

$$1 \quad -144.*R(23)*W(1)*W(2)-9.*R(46)*W(1)*W(3)$$

$$2 \quad +729.*R(21)*W(1)*W(6)-18.*R(31)*W(2)*W(5)$$

$$3 \quad -9.*R(33)*W(2)*W(6)-9.*R(34)*W(3)*W(6)+162.*R(47)*W(4)*W(5)$$

$$4 \quad +2592.*R(3)*W(4)*W(6)$$





$$A(6,4)=81 \cdot R(4) \cdot W(1) \cdot W(2) + 18 \cdot R(12) \cdot W(1) \cdot W(4) - 9 \cdot R(45) \cdot W(1) \cdot W(2)$$

$$1 \quad -144 \cdot R(22) \cdot W(1) \cdot W(3) + 9 \cdot R(53) \cdot W(2) \cdot W(3)$$

$$2 \quad +729 \cdot R(21) \cdot W(1) \cdot W(5) - 9 \cdot R(33) \cdot W(2) \cdot W(5) - 9 \cdot R(34) \cdot W(3) \cdot W(5)$$

$$3 \quad -18 \cdot R(32) \cdot W(3) \cdot W(6) + 2592 \cdot R(3) \cdot W(4) \cdot W(5)$$

$$4 \quad +162 \cdot R(48) \cdot W(4) \cdot W(6)$$

$$A(1,5)=-6 \cdot R(16) \cdot W(1) \cdot W(2) + 162 \cdot R(5) \cdot W(1) \cdot W(4) + 2 \cdot R(41) \cdot W(1) \cdot W(5)$$

$$1 \quad +R(18) \cdot W(2) \cdot W(2) + R(20) \cdot W(2) \cdot W(3) - 144 \cdot R(23) \cdot W(2) \cdot W(4)$$

$$2 \quad -9 \cdot R(46) \cdot W(3) \cdot W(4) + 9 \cdot R(14) \cdot W(4) \cdot W(4) - 2 \cdot R(26) \cdot W(3) \cdot W(5)$$

$$3 \quad +729 \cdot R(21) \cdot W(4) \cdot W(6)$$

$$A(2,5)=-3 \cdot R(16) \cdot W(1) \cdot W(2) + 2 \cdot R(18) \cdot W(1) \cdot W(2) + R(20) \cdot W(1) \cdot W(3)$$

$$1 \quad -144 \cdot R(23) \cdot W(1) \cdot W(4) - R(30) \cdot W(3) \cdot W(3) - 512 \cdot R(3) \cdot W(2) \cdot W(3)$$

$$2 \quad +9 \cdot R(54) \cdot W(3) \cdot W(4) + 2 \cdot R(50) \cdot W(2) \cdot W(5) + 656 \cdot R(3) \cdot W(3) \cdot W(6)$$

$$3 \quad -18 \cdot R(31) \cdot W(4) \cdot W(5) - 9 \cdot R(33) \cdot W(4) \cdot W(6) - R(36) \cdot W(6) \cdot W(2)$$

$$A(3,5)=R(20) \cdot W(1) \cdot W(2) - 9 \cdot R(46) \cdot W(1) \cdot W(4) - 2 \cdot R(26) \cdot W(1) \cdot W(5)$$

$$1 \quad -256 \cdot R(3) \cdot W(2) \cdot W(2) - 2 \cdot R(30) \cdot W(2) \cdot W(3) + 9 \cdot R(54) \cdot W(2) \cdot W(4)$$

$$2 \quad +2 \cdot R(52) \cdot W(3) \cdot W(5) + 656 \cdot R(3) \cdot W(2) \cdot W(6) - 9 \cdot R(34) \cdot W(4) \cdot W(6)$$

$$3 \quad -2 \cdot R(35) \cdot W(5) \cdot W(6)$$

$$A(4,5)=81 \cdot R(5) \cdot W(1) \cdot W(2) - 144 \cdot R(23) \cdot W(1) \cdot W(2) - 9 \cdot R(46) \cdot W(1) \cdot W(3)$$

$$1 \quad +18 \cdot R(14) \cdot W(1) \cdot W(4) + 729 \cdot R(21) \cdot W(1) \cdot W(6)$$

$$2 \quad +9 \cdot R(54) \cdot W(2) \cdot W(3) - 18 \cdot R(31) \cdot W(2) \cdot W(5) - 9 \cdot R(33) \cdot W(2) \cdot W(6)$$

$$3 \quad -9 \cdot R(34) \cdot W(3) \cdot W(6) + 162 \cdot R(47) \cdot W(4) \cdot W(5)$$

$$4 \quad +2592 \cdot R(3) \cdot W(4) \cdot W(6)$$

$$A(5,5)=B2(5)+B3(1) \cdot AX + R(41) \cdot W(1) \cdot W(2) - 2 \cdot R(26) \cdot W(1) \cdot W(3)$$

$$1 \quad +R(50) \cdot W(2) \cdot W(2) + R(52) \cdot W(3) \cdot W(3) - 18 \cdot R(31) \cdot W(2) \cdot W(4)$$

$$2 \quad +81 \cdot R(47) \cdot W(4) \cdot W(4) - 2 \cdot R(35) \cdot W(3) \cdot W(6) + 3 \cdot R(10) \cdot W(5) \cdot W(2)$$

$$3 \quad +R(37) \cdot W(6) \cdot W(2)$$

$$A(6,5)=729 \cdot R(21) \cdot W(1) \cdot W(4) + 656 \cdot R(3) \cdot W(2) \cdot W(3)$$

$$1 \quad -9 \cdot R(33) \cdot W(2) \cdot W(4) - 9 \cdot R(34) \cdot W(3) \cdot W(4) + 1296 \cdot R(3) \cdot W(4) \cdot W(2)$$





$$2 \quad -2 \cdot R(35) \cdot W(3) \cdot W(5) - 2 \cdot R(36) \cdot W(2) \cdot W(6) + 2 \cdot R(37) \cdot W(5) \cdot W(5)$$

$$A(1,6) = -6 \cdot R(15) \cdot W(1) \cdot W(3) + 162 \cdot R(4) \cdot W(1) \cdot W(4) + 2 \cdot R(42) \cdot W(1) \cdot W(6)$$

$$1 \quad + R(17) \cdot W(3) \cdot W(3) + R(19) \cdot W(2) \cdot W(3) - 9 \cdot R(45) \cdot W(2) \cdot W(4)$$

$$2 \quad -144 \cdot R(22) \cdot W(3) \cdot W(4) + 9 \cdot R(12) \cdot W(4) \cdot W(4) - 2 \cdot R(27) \cdot W(2) \cdot W(6)$$

$$3 \quad + 729 \cdot R(21) \cdot W(4) \cdot W(5)$$

$$A(2,6) = R(19) \cdot W(1) \cdot W(3) - 9 \cdot R(45) \cdot W(1) \cdot W(4) - 2 \cdot R(27) \cdot W(1) \cdot W(6)$$

$$1 \quad -256 \cdot R(3) \cdot W(3) \cdot W(3) - 2 \cdot R(29) \cdot W(2) \cdot W(3) + 9 \cdot R(53) \cdot W(3) \cdot W(4)$$

$$2 \quad + 2 \cdot R(51) \cdot W(2) \cdot W(6) + 656 \cdot R(3) \cdot W(2) \cdot W(5) - 9 \cdot R(33) \cdot W(4) \cdot W(5)$$

$$3 \quad -2 \cdot R(36) \cdot W(5) \cdot W(6)$$

$$A(3,6) = -3 \cdot R(15) \cdot W(1) \cdot W(2) + R(19) \cdot W(1) \cdot W(2) + 2 \cdot R(17) \cdot W(1) \cdot W(3)$$

$$1 \quad -144 \cdot R(22) \cdot W(1) \cdot W(4) - R(29) \cdot W(2) \cdot W(3) - 512 \cdot R(3) \cdot W(2) \cdot W(3)$$

$$2 \quad + 9 \cdot R(53) \cdot W(2) \cdot W(4) + 2 \cdot R(49) \cdot W(3) \cdot W(6) + 656 \cdot R(3) \cdot W(2) \cdot W(5)$$

$$3 \quad -18 \cdot R(32) \cdot W(4) \cdot W(6) - 9 \cdot R(34) \cdot W(4) \cdot W(5) - R(35) \cdot W(5) \cdot W(5)$$

$$A(4,6) = 81 \cdot R(4) \cdot W(1) \cdot W(2) - 9 \cdot R(45) \cdot W(1) \cdot W(2) - 144 \cdot R(22) \cdot W(1) \cdot W(3)$$

$$1 \quad + 18 \cdot R(12) \cdot W(1) \cdot W(4) + 729 \cdot R(21) \cdot W(1) \cdot W(5)$$

$$2 \quad + 9 \cdot R(53) \cdot W(2) \cdot W(3) - 18 \cdot R(32) \cdot W(2) \cdot W(6) - 9 \cdot R(33) \cdot W(2) \cdot W(5)$$

$$3 \quad -9 \cdot R(34) \cdot W(3) \cdot W(5) + 162 \cdot R(48) \cdot W(4) \cdot W(6)$$

$$4 \quad + 2592 \cdot R(3) \cdot W(4) \cdot W(5)$$

$$A(5,6) = 729 \cdot R(21) \cdot W(1) \cdot W(4) + 656 \cdot R(3) \cdot W(2) \cdot W(3)$$

$$1 \quad -9 \cdot R(33) \cdot W(2) \cdot W(4) - 9 \cdot R(34) \cdot W(3) \cdot W(4) + 1296 \cdot R(3) \cdot W(4) \cdot W(4)$$

$$2 \quad -2 \cdot R(36) \cdot W(2) \cdot W(6) - 2 \cdot R(35) \cdot W(3) \cdot W(5) + 2 \cdot R(37) \cdot W(5) \cdot W(6)$$

$$A(6,6) = B2(3) + B3(3) \cdot AX + R(42) \cdot W(1) \cdot W(2) - 2 \cdot R(27) \cdot W(1) \cdot W(2)$$

$$1 \quad + R(51) \cdot W(2) \cdot W(2) + R(49) \cdot W(3) \cdot W(2) - 18 \cdot R(32) \cdot W(3) \cdot W(4)$$

$$2 \quad + 81 \cdot R(48) \cdot W(4) \cdot W(2) - 2 \cdot R(36) \cdot W(2) \cdot W(5) + R(37) \cdot W(5) \cdot W(2)$$

$$3 \quad + 3 \cdot R(8) \cdot W(6) \cdot W(2)$$

$$D1 = B1(1) - B2(1) \cdot W(1) - B3(1) \cdot AX \cdot W(1) - R(6) \cdot W(1) \cdot W(1)$$

$$1 \quad + 3 \cdot R(2) \cdot W(1) \cdot W(2) + 3 \cdot R(1) \cdot W(1) \cdot W(3) - R(38) \cdot W(1) \cdot W(2) \cdot W(2)$$

$$2 \quad - R(39) \cdot W(1) \cdot W(3) \cdot W(2) - 32 \cdot R(3) \cdot W(1) \cdot W(2) \cdot W(3)$$



$$\begin{aligned}
& 3 \quad -81 \cdot R(21) \cdot W(1) \cdot W(4) \cdot W(5) + 18 \cdot R(11) \cdot W(1) \cdot W(2) \cdot W(4) \\
& 4 \quad + 18 \cdot R(13) \cdot W(1) \cdot W(3) \cdot W(4) + R(24) \cdot W(2) \cdot W(3) \cdot W(4) \\
& 5 \quad + R(25) \cdot W(2) \cdot W(3) \cdot W(4) - R(40) \cdot W(2) \cdot W(3) \cdot W(4) \\
& 6 \quad + 6 \cdot R(16) \cdot W(1) \cdot W(2) \cdot W(5) + 6 \cdot R(15) \cdot W(1) \cdot W(3) \cdot W(5) \\
& 7 \quad - 162 \cdot R(5) \cdot W(1) \cdot W(4) \cdot W(5) - 162 \cdot R(4) \cdot W(1) \cdot W(4) \cdot W(5)
\end{aligned}$$

$$E1 = -R(41) \cdot W(1) \cdot W(5) \cdot W(6) - R(42) \cdot W(1) \cdot W(5) \cdot W(6) - R(18) \cdot W(2) \cdot W(3) \cdot W(5)$$

$$\begin{aligned}
& 1 \quad -R(17) \cdot W(3) \cdot W(4) \cdot W(5) - R(20) \cdot W(2) \cdot W(3) \cdot W(5) \\
& 2 \quad -R(19) \cdot W(2) \cdot W(3) \cdot W(6) + 144 \cdot R(23) \cdot W(2) \cdot W(4) \cdot W(5) \\
& 3 \quad + 9 \cdot R(45) \cdot W(2) \cdot W(4) \cdot W(6) + 9 \cdot R(46) \cdot W(3) \cdot W(4) \cdot W(5) \\
& 4 \quad + 144 \cdot R(22) \cdot W(3) \cdot W(4) \cdot W(6) - 9 \cdot R(14) \cdot W(4) \cdot W(5) \cdot W(6) \\
& 5 \quad - 9 \cdot R(12) \cdot W(4) \cdot W(5) \cdot W(6) + R(27) \cdot W(2) \cdot W(6) \cdot W(5) \\
& 6 \quad + R(26) \cdot W(3) \cdot W(5) \cdot W(6) - 729 \cdot R(21) \cdot W(4) \cdot W(5) \cdot W(6)
\end{aligned}$$

$$D2 = B1(2) - B2(4) \cdot W(2) - B3(1) \cdot AX \cdot W(2) + R(2) \cdot W(1) \cdot W(2)$$

$$\begin{aligned}
& 1 \quad -R(38) \cdot W(1) \cdot W(2) \cdot W(3) - 16 \cdot R(3) \cdot W(1) \cdot W(2) \cdot W(3) \\
& 2 \quad + 9 \cdot R(11D) \cdot W(1) \cdot W(2) \cdot W(4) + R(25) \cdot W(1) \cdot W(3) \cdot W(4) \\
& 3 \quad + 2 \cdot R(24) \cdot W(1) \cdot W(2) \cdot W(3) - R(40) \cdot W(1) \cdot W(3) \cdot W(4) \\
& 4 \quad - R(9) \cdot W(2) \cdot W(3) \cdot W(4) - R(28) \cdot W(2) \cdot W(3) \cdot W(4) - R(43) \cdot W(2) \cdot W(4) \cdot W(5) \\
& 5 \quad + 243 \cdot R(1) \cdot W(2) \cdot W(3) \cdot W(4) + 3 \cdot R(16) \cdot W(1) \cdot W(2) \cdot W(5) \\
& 6 \quad - 2 \cdot R(18D) \cdot W(1) \cdot W(2) \cdot W(5) - R(20) \cdot W(1) \cdot W(3) \cdot W(5) \\
& 7 \quad - R(19) \cdot W(1) \cdot W(3) \cdot W(6) + 144 \cdot R(23) \cdot W(1) \cdot W(4) \cdot W(5)
\end{aligned}$$

$$E2 = 9 \cdot R(45) \cdot W(1) \cdot W(4) \cdot W(6) + R(27) \cdot W(1) \cdot W(6) \cdot W(5)$$

$$\begin{aligned}
& 1 \quad + R(30) \cdot W(3) \cdot W(4) \cdot W(5) + 256 \cdot R(3) \cdot W(3) \cdot W(4) \cdot W(5) \\
& 2 \quad + 512 \cdot R(3) \cdot W(2) \cdot W(3) \cdot W(5) + 2 \cdot R(29) \cdot W(2) \cdot W(3) \cdot W(6) \\
& 3 \quad - 9 \cdot R(54) \cdot W(3) \cdot W(4) \cdot W(5) - 9 \cdot R(53) \cdot W(3) \cdot W(4) \cdot W(6) \\
& 4 \quad - R(50) \cdot W(2) \cdot W(5) \cdot W(6) - R(51) \cdot W(2) \cdot W(6) \cdot W(5) \\
& 5 \quad - 656 \cdot R(3) \cdot W(3) \cdot W(5) \cdot W(6) + 9 \cdot R(31) \cdot W(4) \cdot W(5) \cdot W(6) \\
& 6 \quad + 9 \cdot R(33) \cdot W(4) \cdot W(5) \cdot W(6) + R(36) \cdot W(5) \cdot W(6) \cdot W(5)
\end{aligned}$$

$$D3 = B1(2) - B2(2) \cdot W(3) - B3(2) \cdot AX \cdot W(3) + R(1) \cdot W(1) \cdot W(2)$$





$$\begin{aligned}
& 1 \quad -16 \cdot R(3) \cdot W(1) \cdot W(2) - R(39) \cdot W(1) \cdot W(2) \cdot W(3) \\
& 2 \quad +9 \cdot R(13) \cdot W(1) \cdot W(2) \cdot W(4) + R(24) \cdot W(1) \cdot W(2) \cdot W(4) \\
& 3 \quad +2 \cdot R(25) \cdot W(1) \cdot W(2) \cdot W(3) - R(40) \cdot W(1) \cdot W(2) \cdot W(4) \\
& 4 \quad -R(28) \cdot W(2) \cdot W(3) - R(7) \cdot W(3) \cdot W(4) + 243 \cdot R(2) \cdot W(3) \cdot W(4) \\
& 5 \quad -9 \cdot R(44) \cdot W(3) \cdot W(4) + 3 \cdot R(15) \cdot W(1) \cdot W(2) \cdot W(6) \\
& 6 \quad -R(20) \cdot W(1) \cdot W(2) \cdot W(5) - R(19) \cdot W(1) \cdot W(2) \cdot W(6) \\
& 7 \quad -2 \cdot R(17) \cdot W(1) \cdot W(3) \cdot W(6) + 9 \cdot R(46) \cdot W(1) \cdot W(4) \cdot W(5)
\end{aligned}$$

$$E3 = 144 \cdot R(22) \cdot W(1) \cdot W(4) \cdot W(6) + R(26) \cdot W(1) \cdot W(5) \cdot W(6)$$

$$\begin{aligned}
& 1 \quad +256 \cdot R(3) \cdot W(2) \cdot W(5) + R(29) \cdot W(2) \cdot W(3) \cdot W(6) \\
& 2 \quad +2 \cdot R(30) \cdot W(2) \cdot W(3) \cdot W(5) + 512 \cdot R(3) \cdot W(2) \cdot W(3) \cdot W(6) \\
& 3 \quad -9 \cdot R(54) \cdot W(2) \cdot W(4) \cdot W(5) - 9 \cdot R(53) \cdot W(2) \cdot W(4) \cdot W(6) \\
& 4 \quad -R(52) \cdot W(3) \cdot W(5) - R(49) \cdot W(3) \cdot W(6) \\
& 5 \quad -656 \cdot R(3) \cdot W(2) \cdot W(5) \cdot W(6) + 9 \cdot R(32) \cdot W(4) \cdot W(6) \\
& 6 \quad +9 \cdot R(34) \cdot W(4) \cdot W(5) \cdot W(6) + R(35) \cdot W(5) \cdot W(6)
\end{aligned}$$

$$D4 = B1(4) - 81 \cdot B2(1) \cdot W(4) - B3(2) \cdot AX \cdot W(4) + 9 \cdot R(11) \cdot W(1) \cdot W(2) \cdot W(2)$$

$$\begin{aligned}
& 1 \quad +9 \cdot R(13) \cdot W(1) \cdot W(2) \cdot W(3) - 81 \cdot R(21) \cdot W(1) \cdot W(2) \cdot W(4) \\
& 2 \quad -R(40) \cdot W(1) \cdot W(2) \cdot W(3) + 81 \cdot R(1) \cdot W(2) \cdot W(3) + 81 \cdot R(2) \cdot W(3) \cdot W(3) \\
& 3 \quad -9 \cdot R(43) \cdot W(2) \cdot W(4) - 81 \cdot R(6) \cdot W(4) - 9 \cdot R(44) \cdot W(3) \cdot W(4) \\
& 4 \quad -81 \cdot R(5) \cdot W(1) \cdot W(2) \cdot W(5) - 81 \cdot R(4) \cdot W(1) \cdot W(2) \cdot W(6) \\
& 5 \quad +144 \cdot R(23) \cdot W(1) \cdot W(2) \cdot W(5) + 9 \cdot R(45) \cdot W(1) \cdot W(2) \cdot W(6) \\
& 6 \quad +9 \cdot R(46) \cdot W(1) \cdot W(3) \cdot W(5) + 144 \cdot R(22) \cdot W(1) \cdot W(3) \cdot W(6)
\end{aligned}$$

$$F4 = -18 \cdot R(14) \cdot W(1) \cdot W(4) \cdot W(5) - 18 \cdot R(12) \cdot W(1) \cdot W(4) \cdot W(6)$$

$$\begin{aligned}
& 1 \quad -729 \cdot R(21) \cdot W(1) \cdot W(5) \cdot W(6) - 9 \cdot R(54) \cdot W(2) \cdot W(3) \cdot W(5) \\
& 2 \quad -9 \cdot R(53) \cdot W(2) \cdot W(3) \cdot W(6) + 9 \cdot R(31) \cdot W(2) \cdot W(5) \cdot W(6) \\
& 3 \quad +9 \cdot R(32) \cdot W(3) \cdot W(6) + 9 \cdot R(33) \cdot W(2) \cdot W(5) \cdot W(6) \\
& 4 \quad +9 \cdot R(34) \cdot W(3) \cdot W(5) \cdot W(6) - 81 \cdot R(47) \cdot W(4) \cdot W(5) \cdot W(6) \\
& 5 \quad -81 \cdot R(48) \cdot W(4) \cdot W(6) - 2592 \cdot R(3) \cdot W(4) \cdot W(5) \cdot W(6)
\end{aligned}$$

$$D5 = B1(3) - B2(5) \cdot W(5) - B3(1) \cdot AX \cdot W(5) + 3 \cdot R(16) \cdot W(1) \cdot W(2) \cdot W(2)$$





$$\begin{aligned}
1 & -81 \cdot R(5) \cdot W(1) \cdot W(4) - R(18) \cdot W(1) \cdot W(2) \cdot W(3) \\
2 & -9 \cdot R(14) \cdot W(1) \cdot W(4) + 144 \cdot R(23) \cdot W(1) \cdot W(2) \cdot W(4) \\
3 & +9 \cdot R(46) \cdot W(1) \cdot W(3) \cdot W(4) + 256 \cdot R(3) \cdot W(2) \cdot W(3) \\
4 & +R(30) \cdot W(2) \cdot W(3) - 9 \cdot R(54) \cdot W(2) \cdot W(3) \cdot W(4) \\
5 & -R(41) \cdot W(1) \cdot W(5) + 2 \cdot R(26) \cdot W(1) \cdot W(3) \cdot W(5) \\
6 & -729 \cdot R(21) \cdot W(1) \cdot W(4) \cdot W(6) - R(50) \cdot W(2) \cdot W(5)
\end{aligned}$$

$$E5 = -R(52) \cdot W(3) \cdot W(5) - 656 \cdot R(3) \cdot W(2) \cdot W(3) \cdot W(6)$$

$$\begin{aligned}
1 & +18 \cdot R(31) \cdot W(2) \cdot W(4) \cdot W(5) + 9 \cdot R(33) \cdot W(2) \cdot W(4) \cdot W(6) \\
2 & +9 \cdot R(34) \cdot W(3) \cdot W(4) \cdot W(6) - 81 \cdot R(47) \cdot W(4) \cdot W(5) \\
3 & -1296 \cdot R(3) \cdot W(4) \cdot W(6) + R(36) \cdot W(2) \cdot W(6) \\
4 & +2 \cdot R(35) \cdot W(3) \cdot W(5) \cdot W(6) - R(10) \cdot W(5) \cdot W(6) - R(37) \cdot W(5) \cdot W(6)
\end{aligned}$$

$$D6 = B1(3) - B2(3) \cdot W(6) - B3(3) \cdot AX \cdot W(6) + 3 \cdot R(15) \cdot W(1) \cdot W(3)$$

$$\begin{aligned}
1 & -81 \cdot R(4) \cdot W(1) \cdot W(4) - R(17) \cdot W(1) \cdot W(3) \cdot W(4) - R(19) \cdot W(1) \cdot W(2) \cdot W(3) \\
2 & -9 \cdot R(12) \cdot W(1) \cdot W(4) + 9 \cdot R(45) \cdot W(1) \cdot W(2) \cdot W(4) \\
3 & +144 \cdot R(22) \cdot W(1) \cdot W(3) \cdot W(4) + R(29) \cdot W(2) \cdot W(3) \\
4 & +256 \cdot R(3) \cdot W(2) \cdot W(3) - 9 \cdot R(53) \cdot W(2) \cdot W(3) \cdot W(4) \\
5 & -R(42) \cdot W(1) \cdot W(6) + 2 \cdot R(27) \cdot W(1) \cdot W(2) \cdot W(6) \\
6 & -729 \cdot R(21) \cdot W(1) \cdot W(4) \cdot W(5) - R(51) \cdot W(2) \cdot W(6)
\end{aligned}$$

$$E6 = -R(49) \cdot W(3) \cdot W(6) - 656 \cdot R(3) \cdot W(2) \cdot W(3) \cdot W(5)$$

$$\begin{aligned}
1 & +9 \cdot R(33) \cdot W(2) \cdot W(4) \cdot W(5) + 9 \cdot R(34) \cdot W(3) \cdot W(4) \cdot W(5) \\
2 & +18 \cdot R(32) \cdot W(3) \cdot W(4) \cdot W(6) - 1296 \cdot R(3) \cdot W(4) \cdot W(5) \\
3 & -81 \cdot R(48) \cdot W(4) \cdot W(6) + R(35) \cdot W(3) \cdot W(5) \\
4 & +2 \cdot R(36) \cdot W(5) \cdot W(6) - R(37) \cdot W(5) \cdot W(6) - R(8) \cdot W(6)
\end{aligned}$$

$$C(1) = D1 + E1$$

$$C(2) = D2 + E2$$

$$C(3) = D3 + E3$$

$$C(4) = D4 + E4$$

$$C(5) = D5 + E5$$



C(6) = D6 + E6

RETURN

END

DATA





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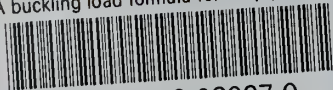
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